Skinning: Real-time Shape Deformation

Direct Skinning Methods and Deformation Primitives

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Variational vs. direct methods

Variational
(numerical optimization)

$$\mathbf{v}' = \arg\min_{\mathbf{x}} \mathrm{E}(\mathbf{x})$$

Direct
(closed-form)

$$\mathbf{v}_i' = \sum_{j=1}^{m} w_{i,j}\, \mathbf{T}_j \mathbf{v}_i$$

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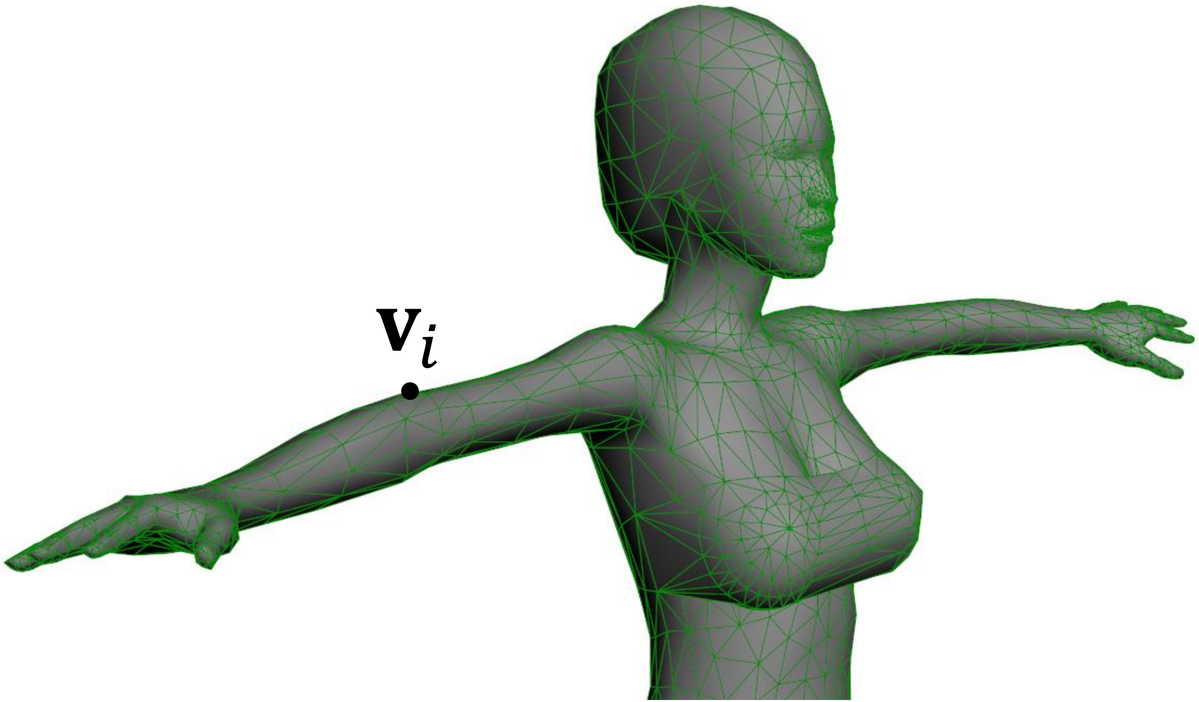
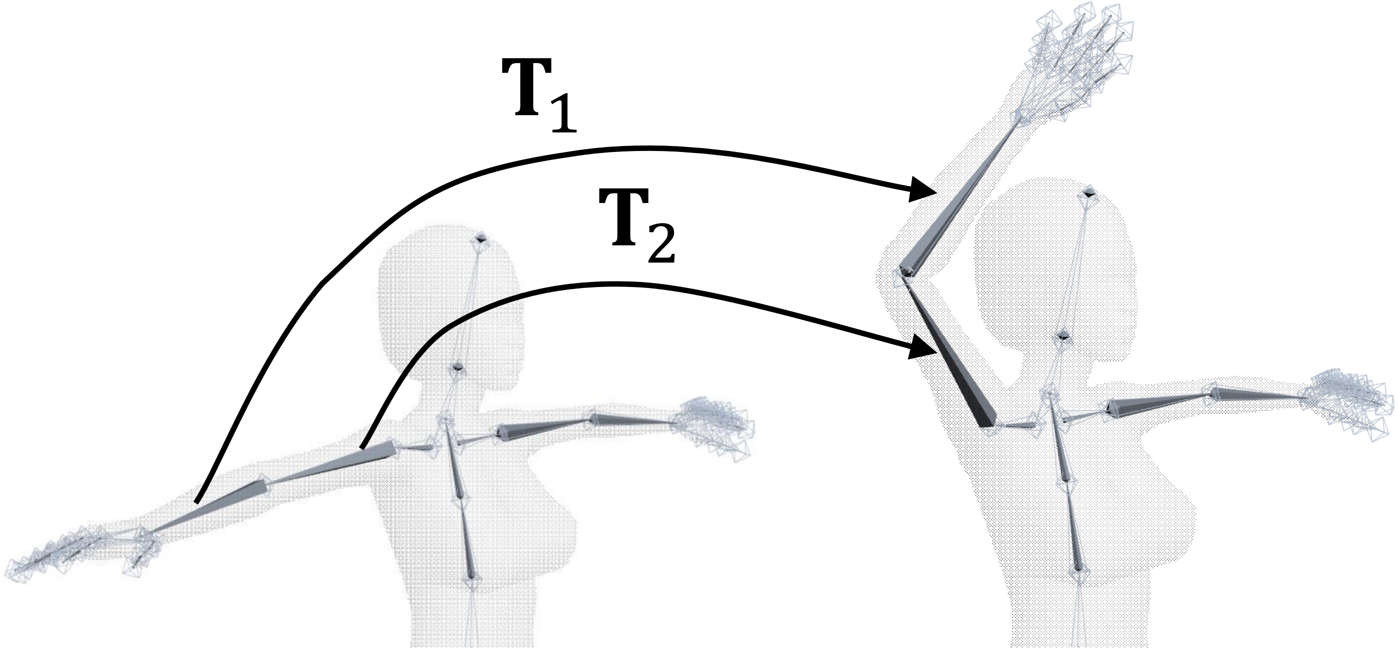
Linear blend skinning: basic setup

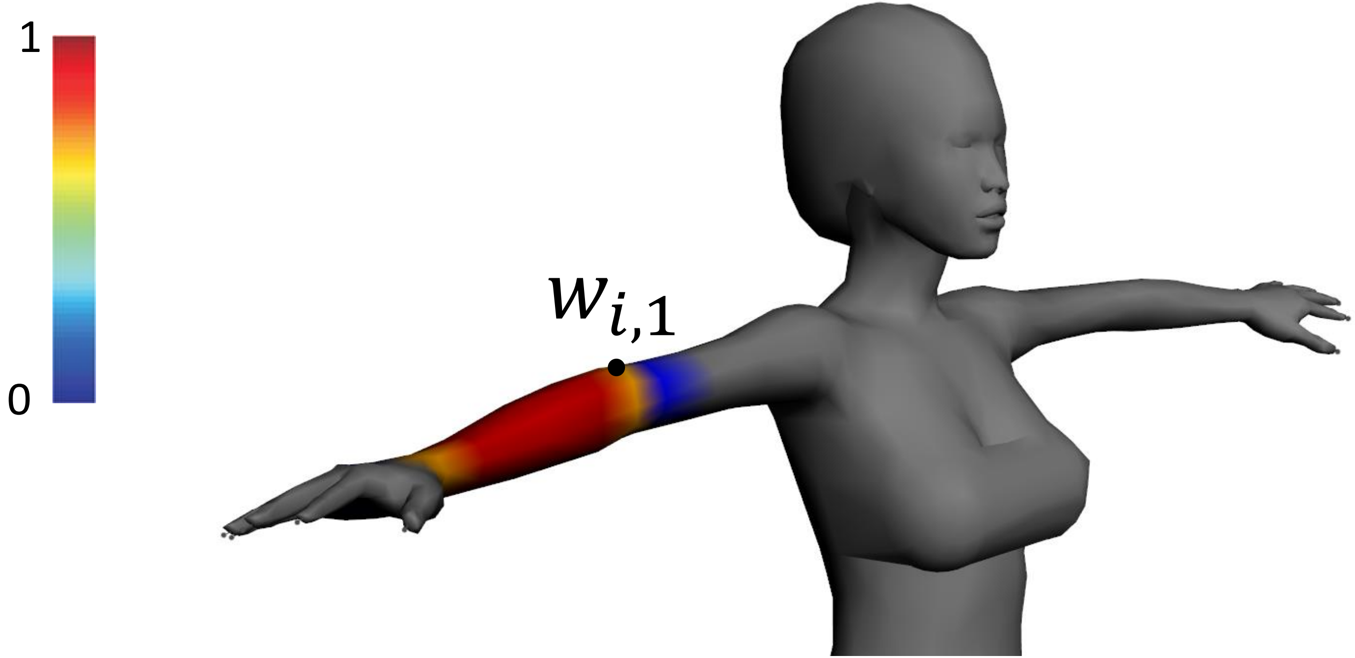
1) Rest pose



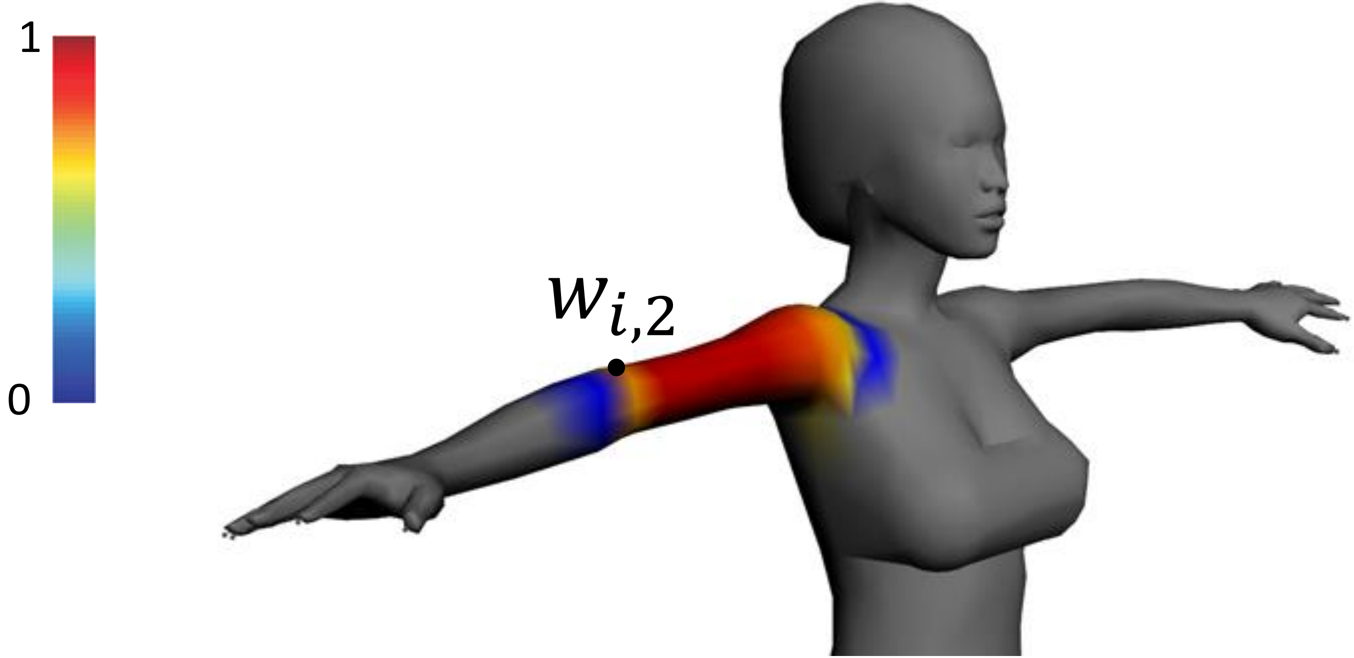
2) Skinning transformations



3) Skinning weights



3) Skinning weights



Linear blend skinning (LBS)

$$\mathbf{v}_i' = \sum_{j=1}^{m} w_{i,j}\, \mathbf{T}_j \mathbf{v}_i = \left(\sum_{j=1}^{m} w_{i,j}\, \mathbf{T}_j \right) \mathbf{v}_i$$

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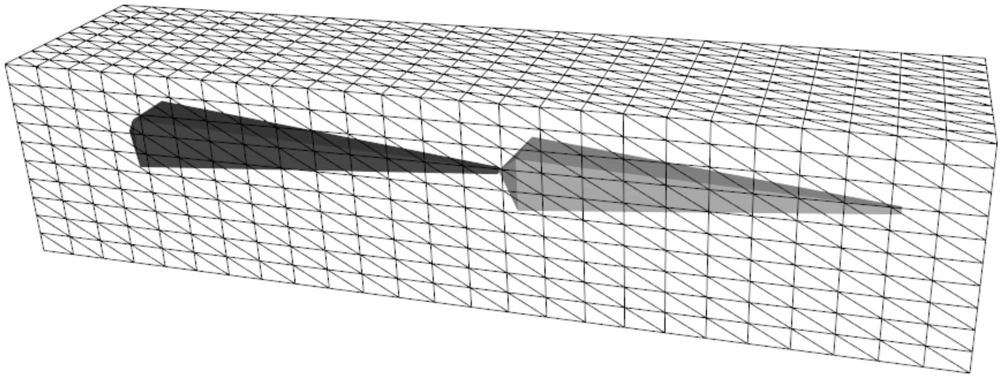
LBS is used widely in the industry

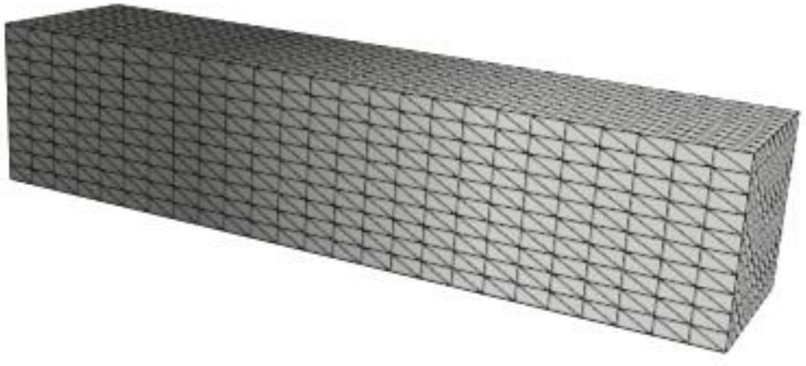
Halo 3

Bolt

LBS: candy-wrapper artifact



LBS: candy-wrapper artifact



Advanced skinning methods

1. Multi-linear methods
2. Nonlinear methods
3. Advanced deformation primitives (deformers)

Multi-linear skinning techniques

Multi-weight enveloping [Wang and Phillips 2002]

Animation Space [Merry et al. 2006]

General linear skinning model

$$\mathbf{v}' = \mathbf{X}\,\mathbf{t}$$

General linear skinning model

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General linear skinning model

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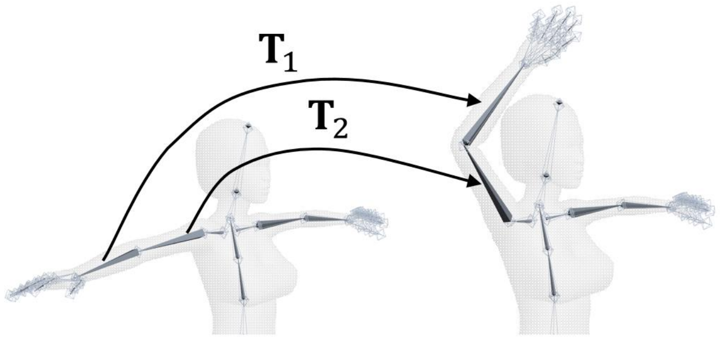
$\mathbf{t}$ stacks skinning transformations [$\mathbf{T}_1, \ldots, \mathbf{T}_m$]

$$\mathbf{t} \in R^{12m}$$



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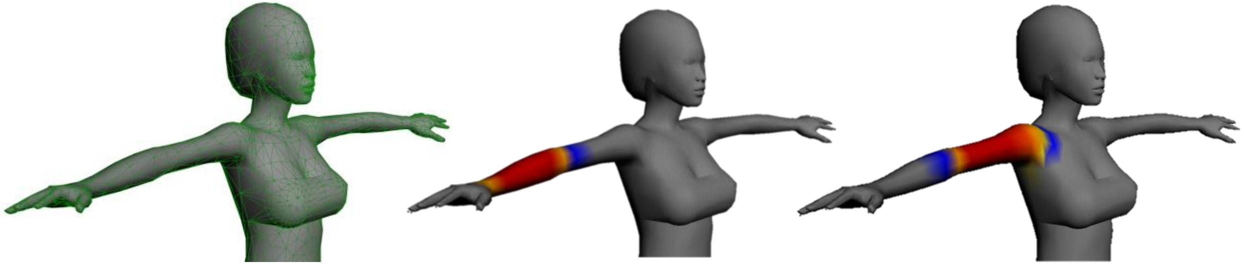
$\mathbf{X} \in R^{3n \times 12m}$

Aggregates rest pose & weights



General linear skinning model

$$\mathbf{v}' = \mathbf{X}\,\mathbf{t}$$

General linear skinning model

$$\mathbf{v}' = \mathbf{X}\,\mathbf{t}$$

The structure of matrix $\mathbf{X}$

$$\mathbf{X} \in R^{3n \times 12m}$$

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_{1,1} & \cdots & \mathbf{X}_{1,m} \\ \vdots & \ddots & \vdots \\ \mathbf{X}_{n,1} & \cdots & \mathbf{X}_{n,m} \end{bmatrix}$$

$$\mathbf{X}_{i,j} \in R^{3 \times 12}$$

LBS in matrix form

$$\mathbf{X}_{i,j}^{\mathrm{LBS}} = [w_{i,j}v_{i,1}\mathbf{I}_3 \quad w_{i,j}v_{i,2}\mathbf{I}_3 \quad w_{i,j}v_{i,3}\mathbf{I}_3 \quad w_{i,j}\mathbf{I}_3]$$

$$\mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{v}_i = \begin{bmatrix} v_{i,1} \\ v_{i,2} \\ v_{i,3} \end{bmatrix}$$

LBS in matrix form

$$\begin{bmatrix} w_{i,j}v_{i,1} & 0 & 0 & w_{i,j}v_{i,2} & 0 & 0 & w_{i,j}v_{i,3} & 0 & 0 & w_{i,j} & 0 & 0 \\ 0 & w_{i,j}v_{i,1} & 0 & 0 & w_{i,j}v_{i,2} & 0 & 0 & w_{i,j}v_{i,3} & 0 & 0 & w_{i,j} & 0 \\ 0 & 0 & w_{i,j}v_{i,1} & 0 & 0 & w_{i,j}v_{i,2} & 0 & 0 & w_{i,j}v_{i,3} & 0 & 0 & w_{i,j} \end{bmatrix}$$

LBS in matrix form

$$\begin{bmatrix} w_{i,j}v_{i,1} & 0 & 0 & w_{i,j}v_{i,2} & 0 & 0 & w_{i,j}v_{i,3} & 0 & 0 & w_{i,j} & 0 & 0 \\ 0 & w_{i,j}v_{i,1} & 0 & 0 & w_{i,j}v_{i,2} & 0 & 0 & w_{i,j}v_{i,3} & 0 & 0 & w_{i,j} & 0 \\ 0 & 0 & w_{i,j}v_{i,1} & 0 & 0 & w_{i,j}v_{i,2} & 0 & 0 & w_{i,j}v_{i,3} & 0 & 0 & w_{i,j} \end{bmatrix}$$

There is just 1 weight (per vertex/bone pair)

Multi-weight Enveloping [Wang and Phillips 2002]

$$\begin{bmatrix} w_{i,j}^{1}v_{i,1} & 0 & 0 & w_{i,j}^{4}v_{i,2} & 0 & 0 & w_{i,j}^{7}v_{i,3} & 0 & 0 & w_{i,j}^{10} & 0 & 0 \\ 0 & w_{i,j}^{2}v_{i,1} & 0 & 0 & w_{i,j}^{5}v_{i,2} & 0 & 0 & w_{i,j}^{8}v_{i,3} & 0 & 0 & w_{i,j}^{11} & 0 \\ 0 & 0 & w_{i,j}^{3}v_{i,1} & 0 & 0 & w_{i,j}^{6}v_{i,2} & 0 & 0 & w_{i,j}^{9}v_{i,3} & 0 & 0 & w_{i,j}^{12} \end{bmatrix}$$

Multi-weight Enveloping [Wang and Phillips 2002]

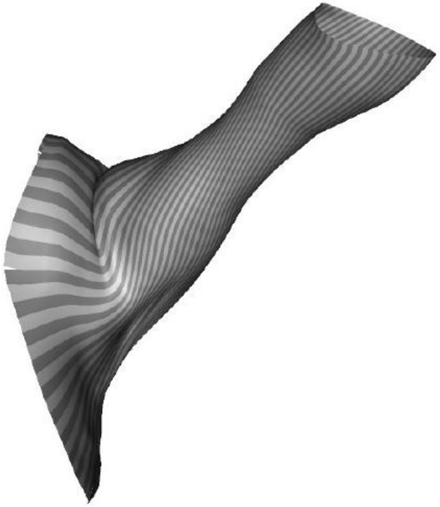
$$\begin{bmatrix} w_{i,j}^1 v_{i,1} & 0 & 0 & w_{i,j}^4 v_{i,2} & 0 & 0 & w_{i,j}^7 v_{i,3} & 0 & 0 & w_{i,j}^{10} & 0 & 0 \\ 0 & w_{i,j}^2 v_{i,1} & 0 & 0 & w_{i,j}^5 v_{i,2} & 0 & 0 & w_{i,j}^8 v_{i,3} & 0 & 0 & w_{i,j}^{11} & 0 \\ 0 & 0 & w_{i,j}^3 v_{i,1} & 0 & 0 & w_{i,j}^6 v_{i,2} & 0 & 0 & w_{i,j}^9 v_{i,3} & 0 & 0 & w_{i,j}^{12} \end{bmatrix}$$

12 different weights (per vertex/bone pair)

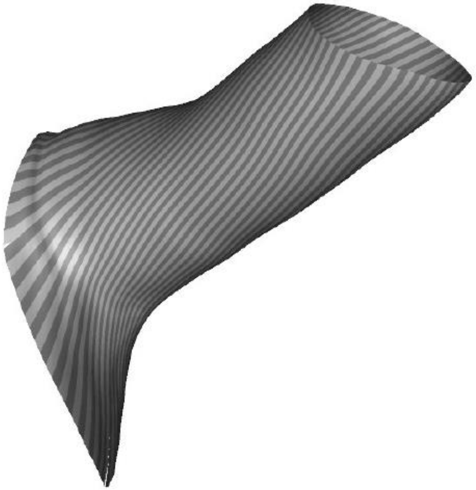
Multi-weight Enveloping [Wang and Phillips 2002]

More powerful

Weights trained from examples



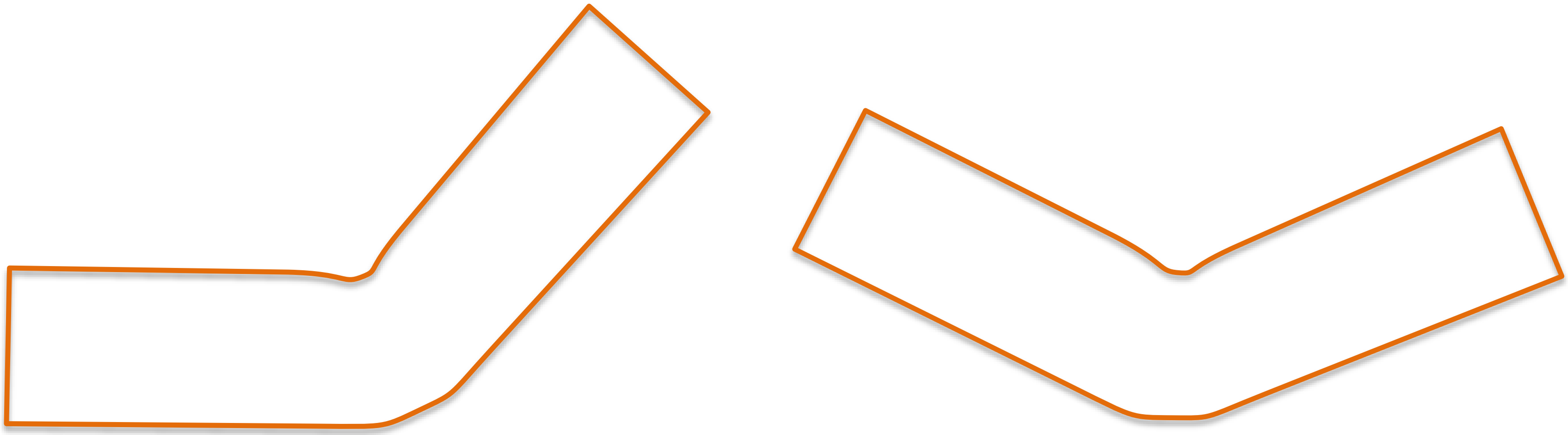
LBS



MWE

Animation Space [Merry et al. 2006]

MWE is “too general” if we want world-space rotation invariance.



Rotation-invariant linear skinning

$$\mathbf{X}_{i,j} = [y_{i,j,1}\mathbf{I}_3 \quad y_{i,j,2}\mathbf{I}_3 \quad y_{i,j,3}\mathbf{I}_3 \quad y_{i,j,4}\mathbf{I}_3]$$

4 weights per vertex/bone pair

Translation invariance: $\displaystyle\sum_{j=1}^{m} y_{i,j,4} = 1$

Rotation-invariant linear skinning

$$\begin{bmatrix} y_{i,j,1} & 0 & 0 & y_{i,j,2} & 0 & 0 & y_{i,j,3} & 0 & 0 & y_{i,j,4} & 0 & 0 \\ 0 & y_{i,j,1} & 0 & 0 & y_{i,j,2} & 0 & 0 & y_{i,j,3} & 0 & 0 & y_{i,j,4} & 0 \\ 0 & 0 & y_{i,j,1} & 0 & 0 & y_{i,j,2} & 0 & 0 & y_{i,j,3} & 0 & 0 & y_{i,j,4} \end{bmatrix}$$

Animation Space [Merry et al. 2006]

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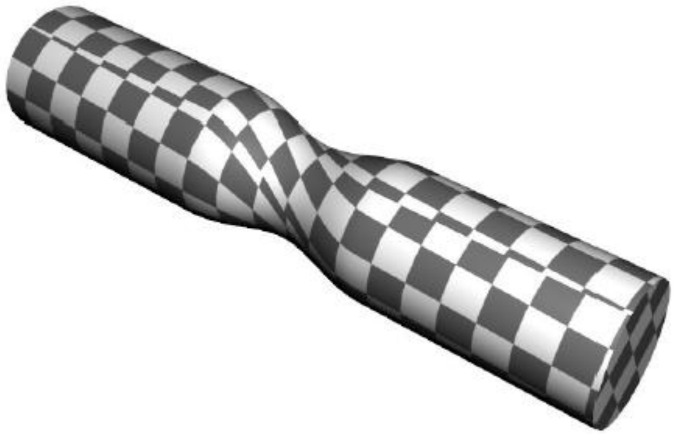
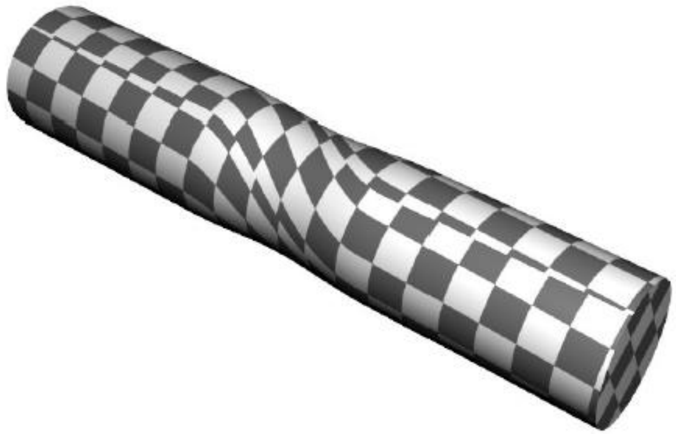


LBS



Animation space

Summary of multi-linear methods

Improve quality by introducing more weights

Nonlinear methods

The LBS formula:

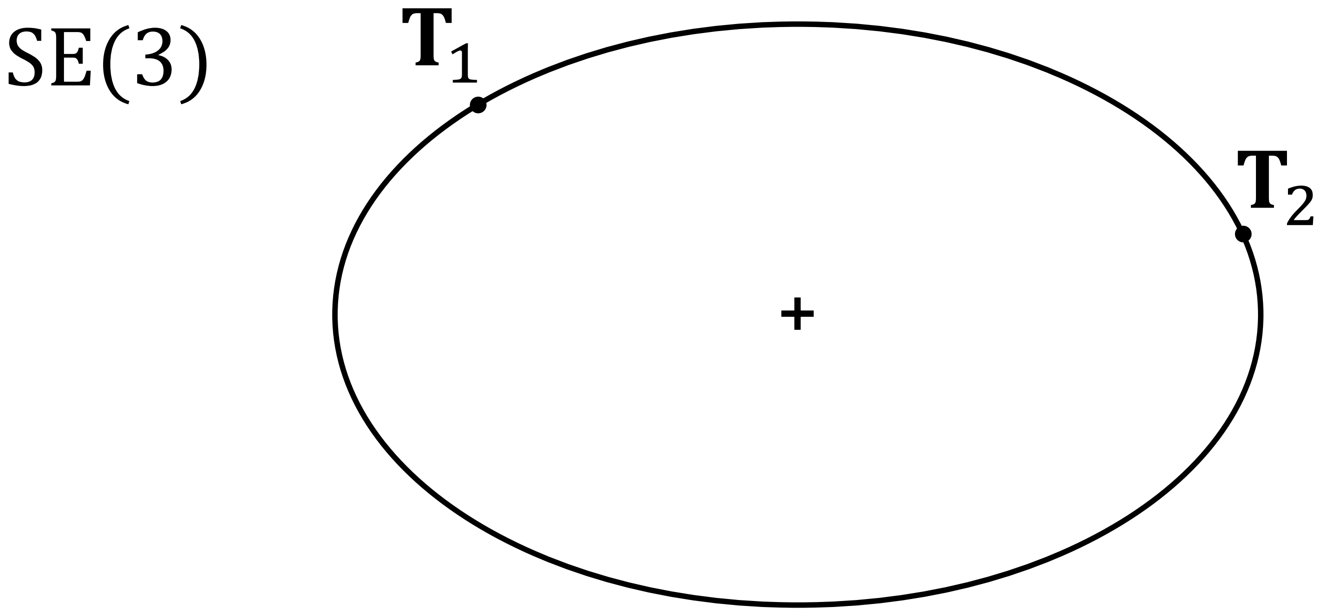
$$\mathbf{v}_i' = \sum_{j=1}^{m} w_{i,j}\,\mathbf{T}_j\mathbf{v}_i = \left(\sum_{j=1}^{m} w_{i,j}\,\mathbf{T}_j\right)\mathbf{v}_i$$

Assume the transformations $\mathbf{T}_j$ are rigid

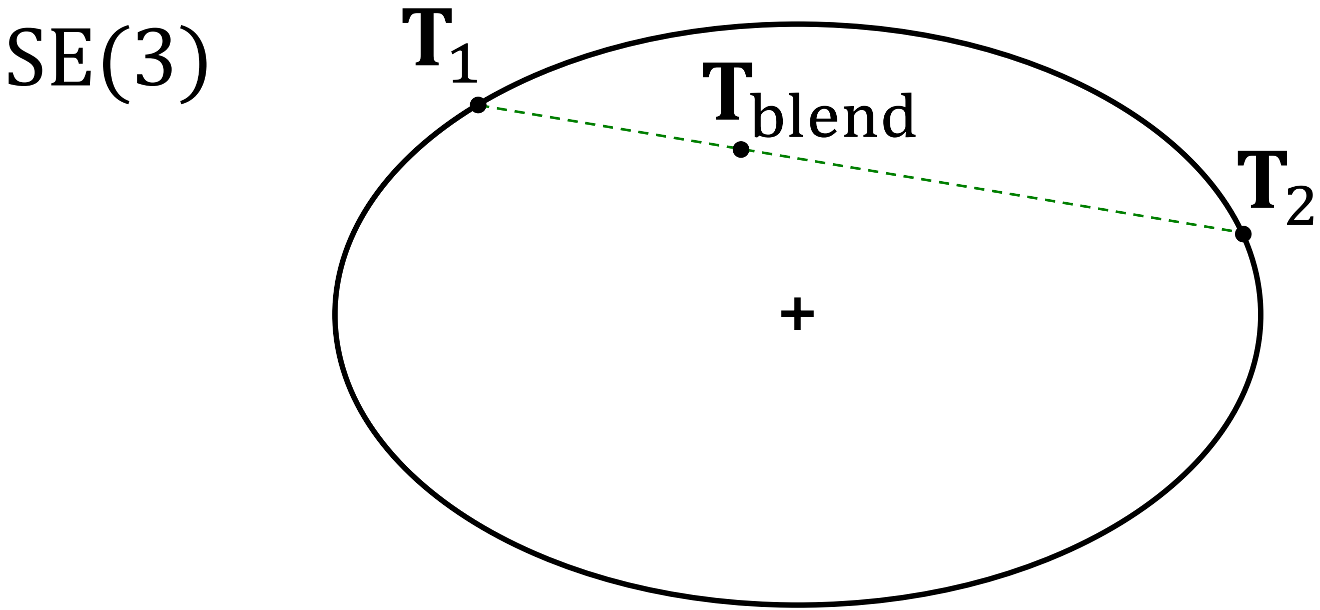
Geometry of linear blending

$\mathrm{SE}(3)$

Geometry of linear blending



Geometry of linear blending



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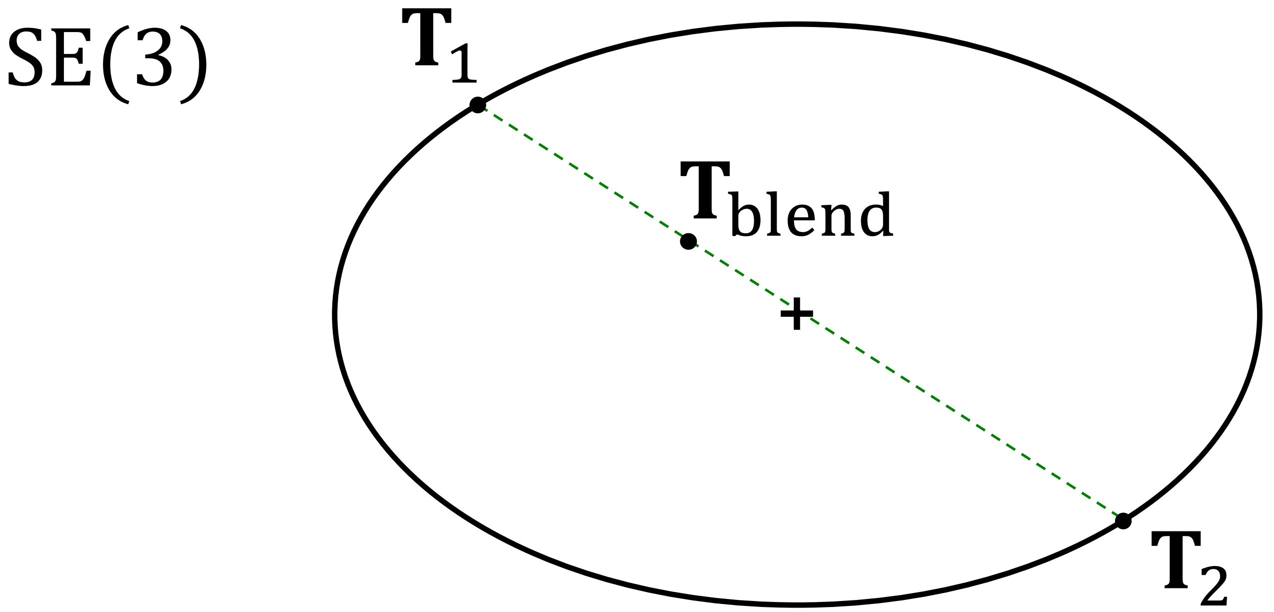
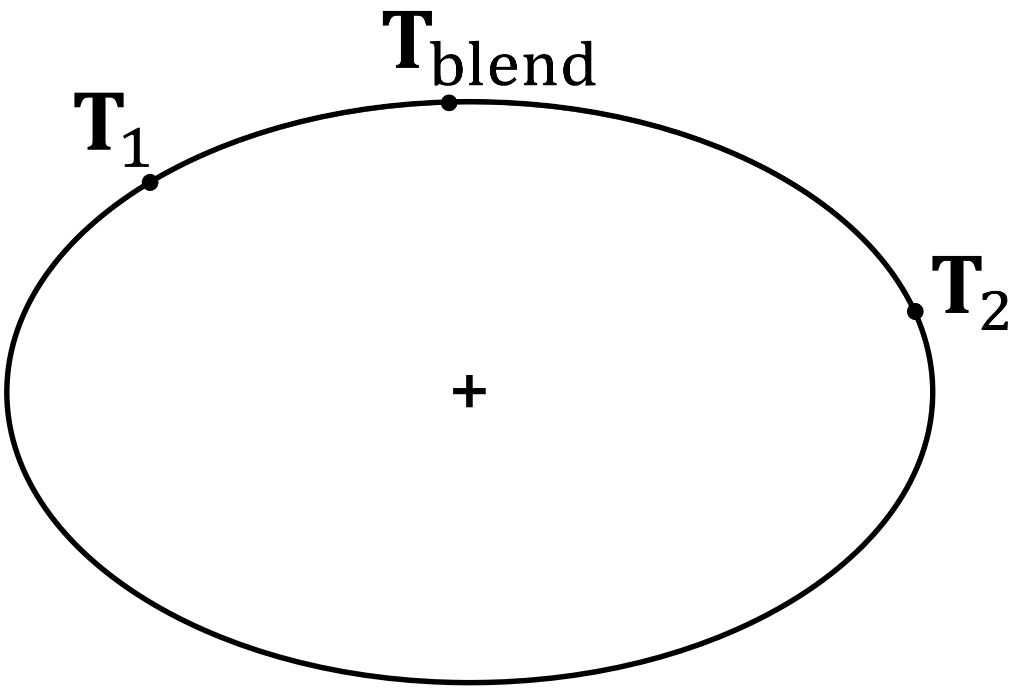


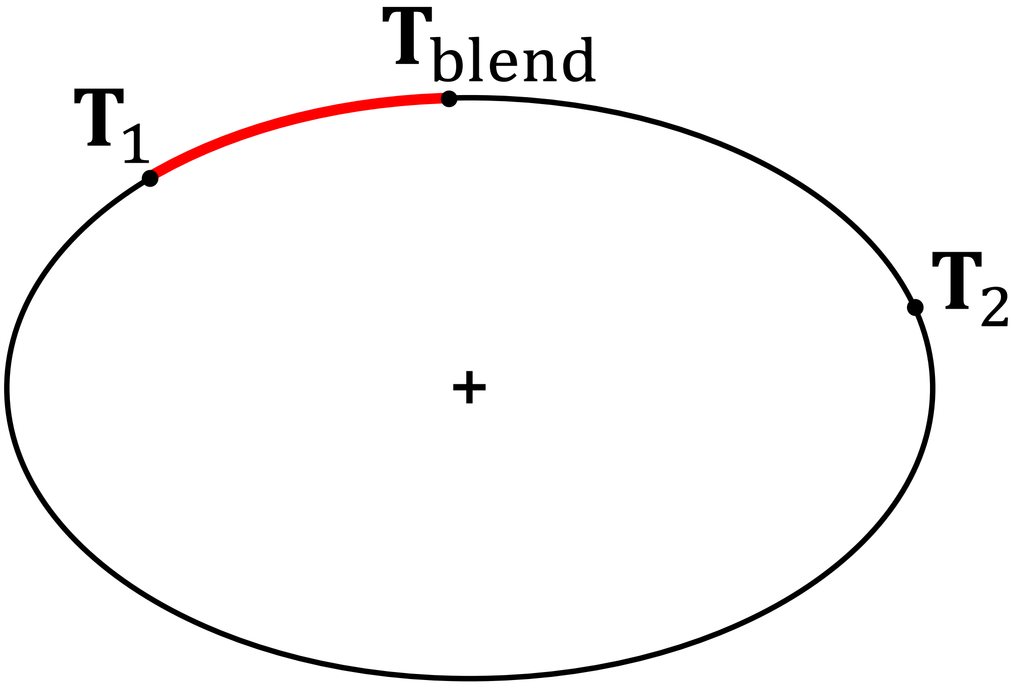
Geometry of linear blending

$$\boldsymbol{R}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \boldsymbol{R}_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

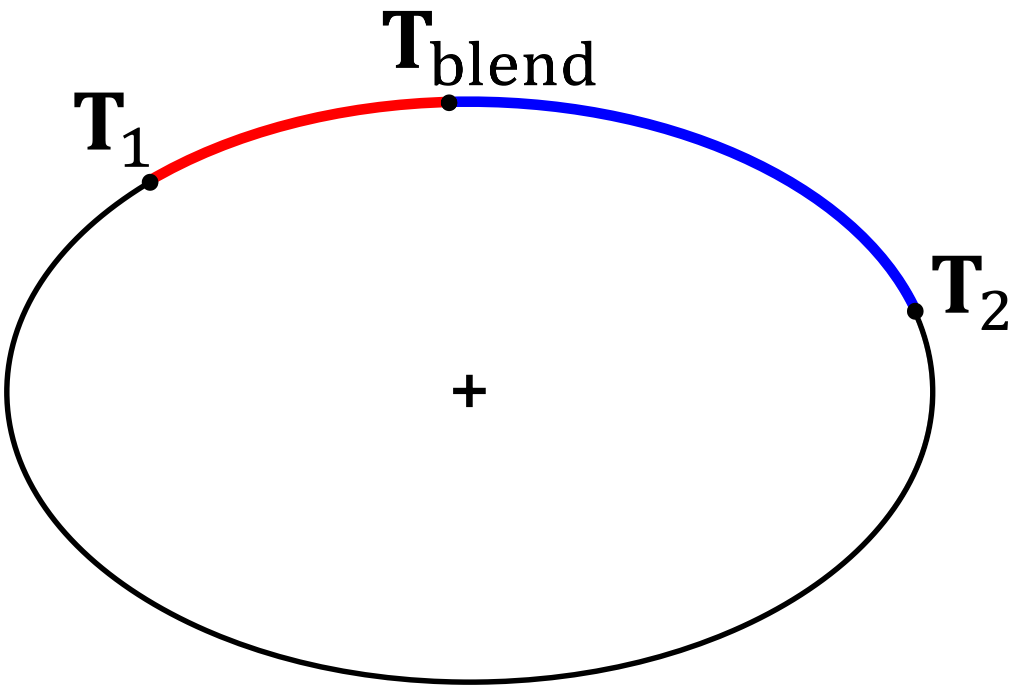
Intrinsic blending



Intrinsic blending



Intrinsic blending



Intrinsic blending using Lie algebras

[Buss and Fillmore 2001, Alexa 2002, Govindu 2004, Rossignac and Vinacua 2011]

$$\arg\min_{\mathbf{X}} \sum_{j=1}^{m} w_j d(\mathbf{X}, \mathbf{T}_j)$$

$$d(\mathbf{X}, \mathbf{Y}) = \|\log(\mathbf{Y}\mathbf{X}^{-1})\|^2$$

Karcher / Frechet mean

Intrinsic blending using Lie algebras

[Buss and Fillmore 2001, Alexa 2002, Govindu 2004, Rossignac and Vinacua 2011]

$$\arg\min_{\mathbf{X}} \sum_{j=1}^{m} w_j d(\mathbf{X}, \mathbf{T}_j)$$

Not a direct method!

$$d(\mathbf{X}, \mathbf{Y}) = \|\log(\mathbf{Y}\mathbf{X}^{-1})\|^2$$

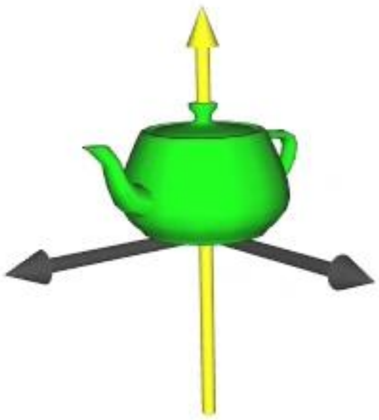
Karcher / Frechet mean

Dual quaternions: approximation of intrinsic averages in SE(3) [Kavan et al. 2008]

$\mathbf{T}_1 \in$ SE(3) can be converted to a unit dual quaternion $\hat{\mathbf{q}} \in \widehat{\mathrm{Q}}_1$

Dual quaternions and rigid motion

$+1.0+0.0i+0.0j+0.0k$



Dual quaternion skinning [Kavan et al. 2008]

One to one correspondence between SE(3) and $\widehat{\mathrm{Q}}_1$?

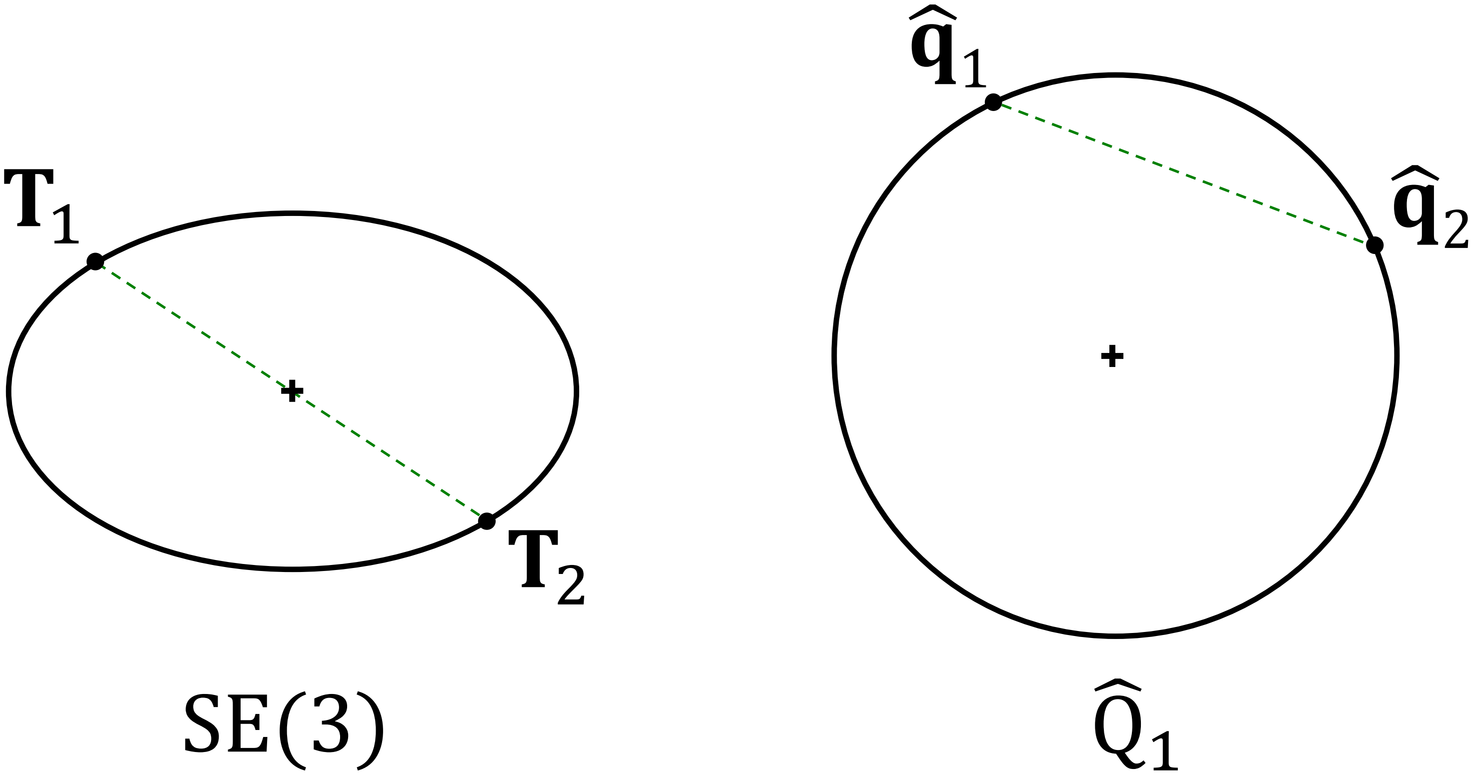
No!

Dual quaternion skinning [Kavan et al. 2008]

$\hat{\mathbf{q}}$ and $-\hat{\mathbf{q}}$ represent the same transformation

$\widehat{\mathrm{Q}}_1$ is a double cover of SE(3)

Double cover visualized

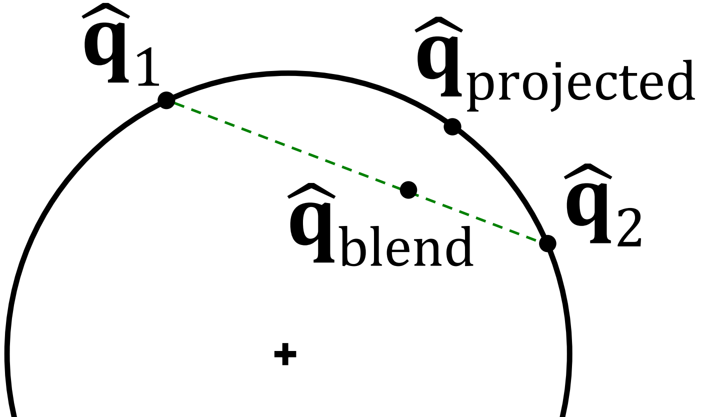


The advantages of dual quaternions

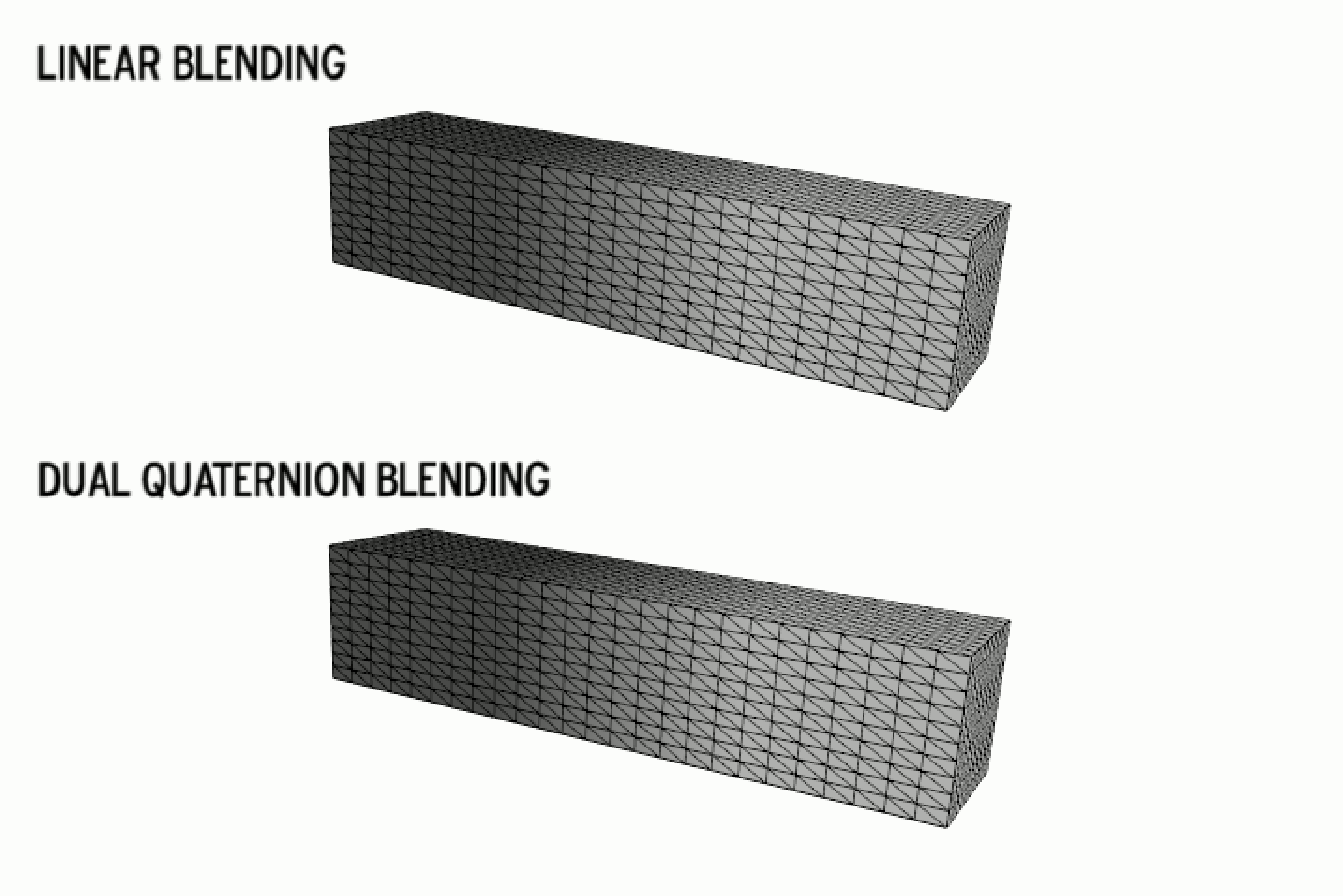
1. $\widehat{\mathrm{Q}}_1$ is “less curved” than SE(3)

2. Fast, closed-form projection on $\widehat{\mathrm{Q}}_1$

Direct!



Dual quaternion skinning [Kavan et al. 2008]



Limitations of dual quaternions

Non-rigid $\mathbf{T}_j$ not directly supported

Open problem: double cover of SL(3)

Non-rigid transformations can be done with two-phase skinning

1. Scaling 2. Rotations

Two-phase skinning

Enhanced for production use by Disney

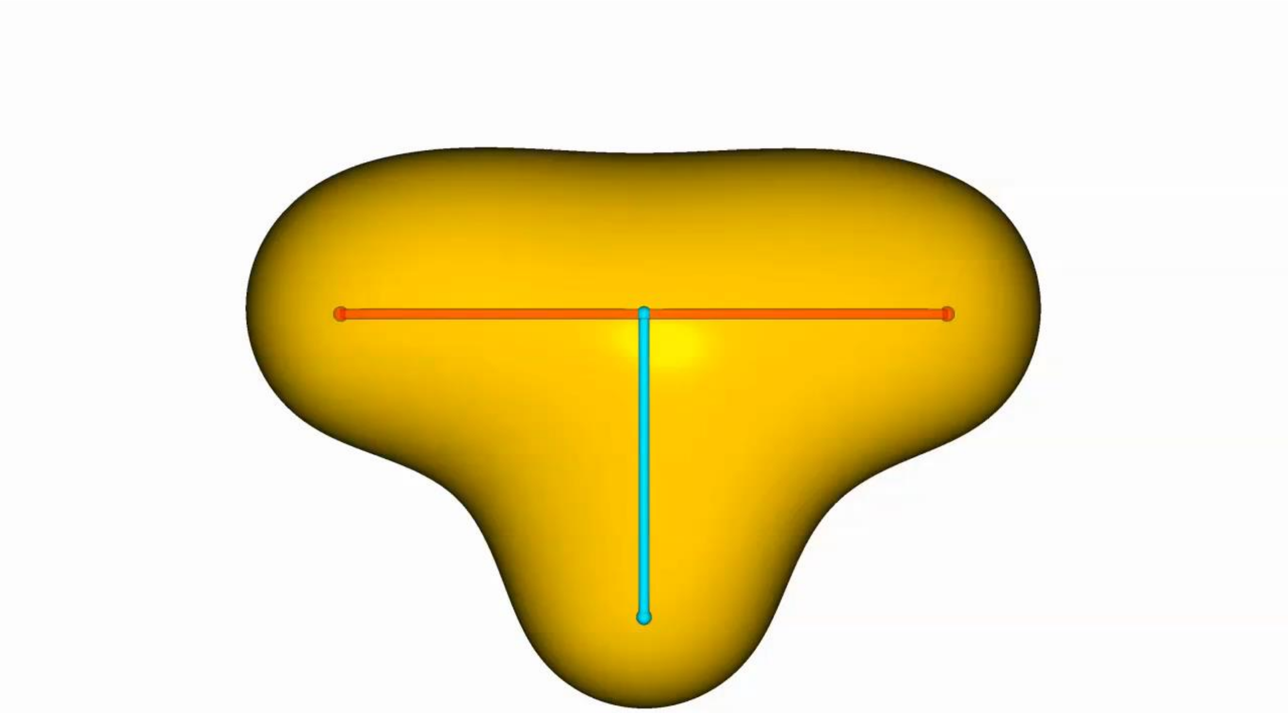
[Lee et al. 2013]

Neither matrices nor quaternions remember multiple revolutions

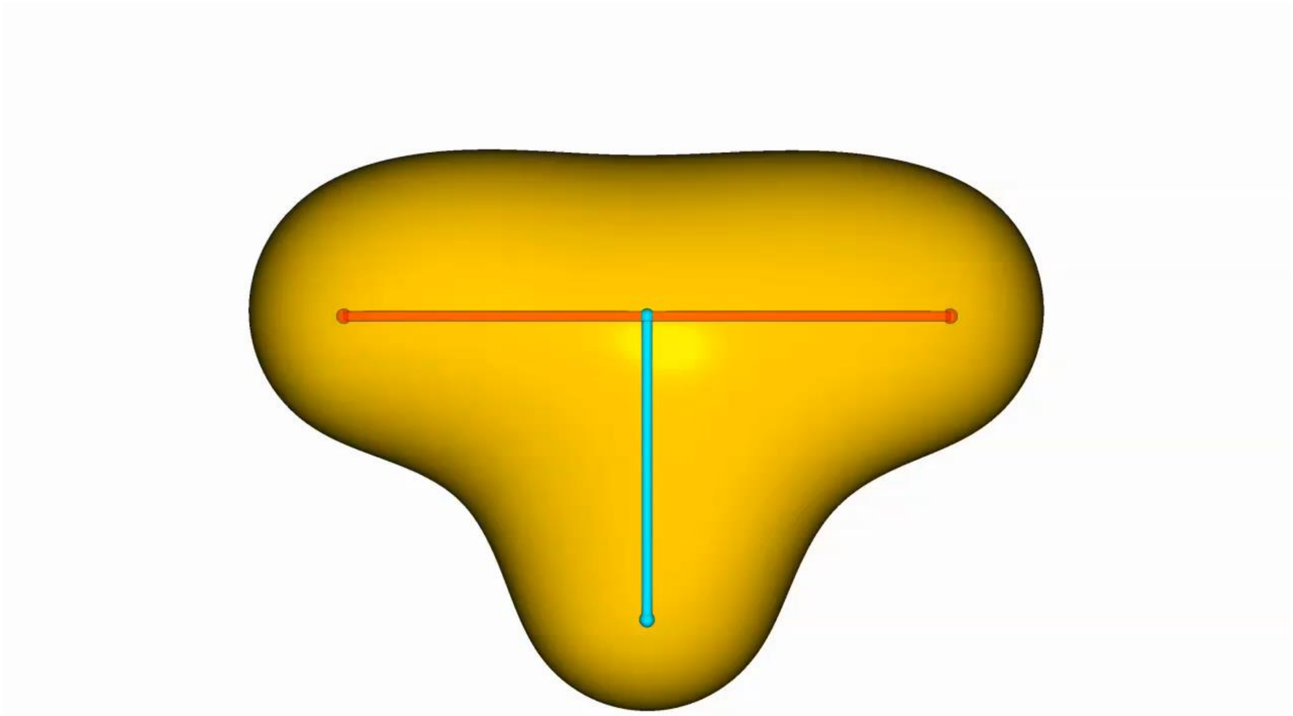
LBS & DQS: shortest path interpolation

Not always desired

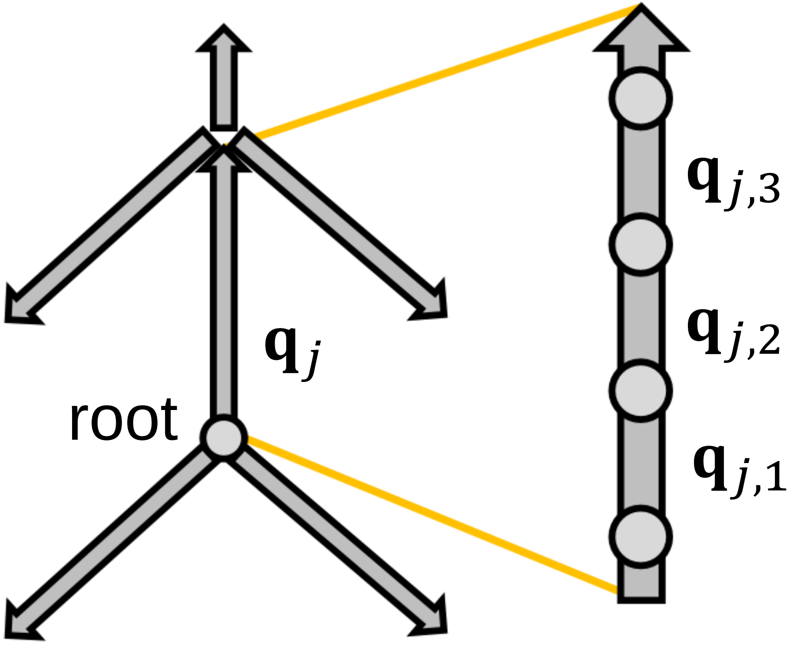
Linear blend skinning



Dual quaternion skinning



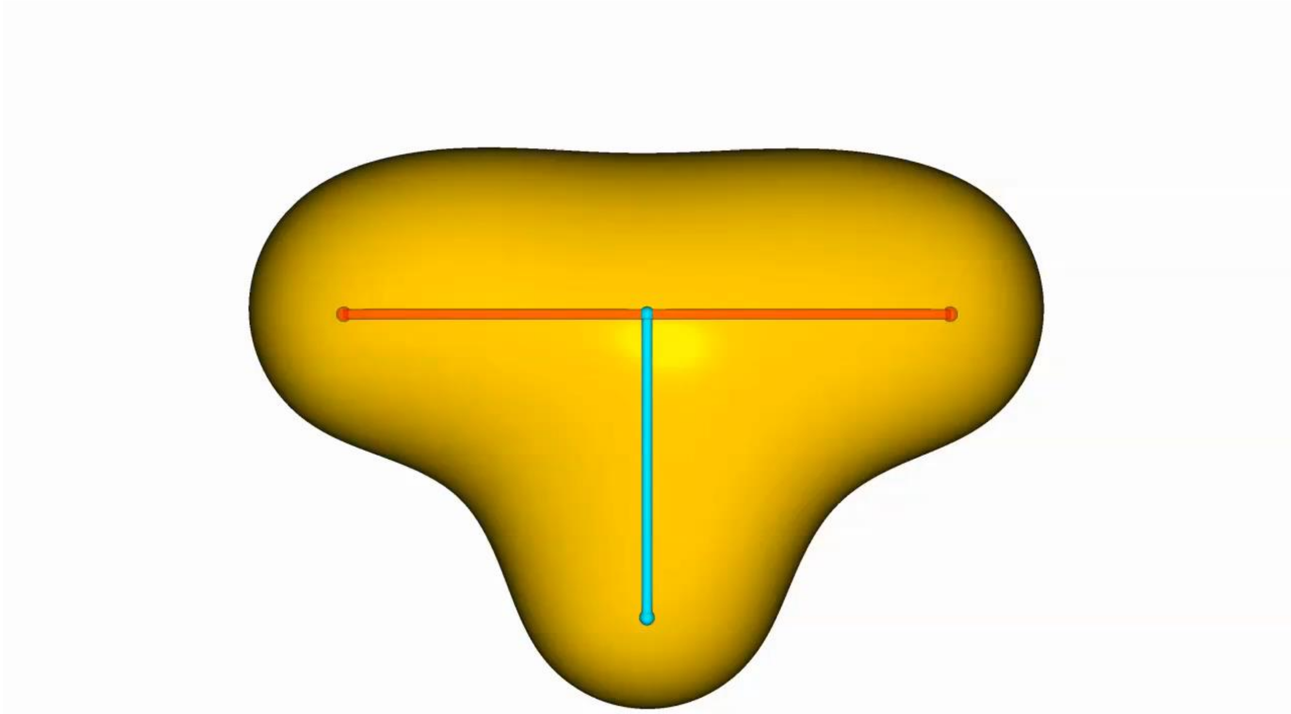
Differential blending [Oztireli et al. 2013]



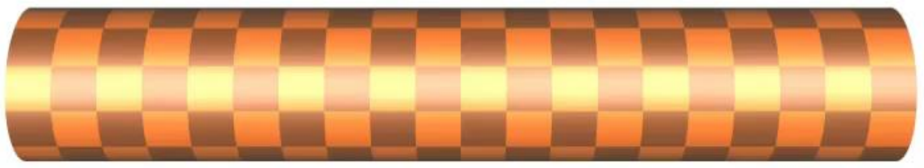
Differential blending [Oztireli et al. 2013]

$$\mathrm{DiffBlend}\left(w_j; \mathbf{q}_j\right) = \prod_{k=1}^{l} \mathrm{Blend}\left(w_j; \mathbf{q}_{j,k}\right)$$

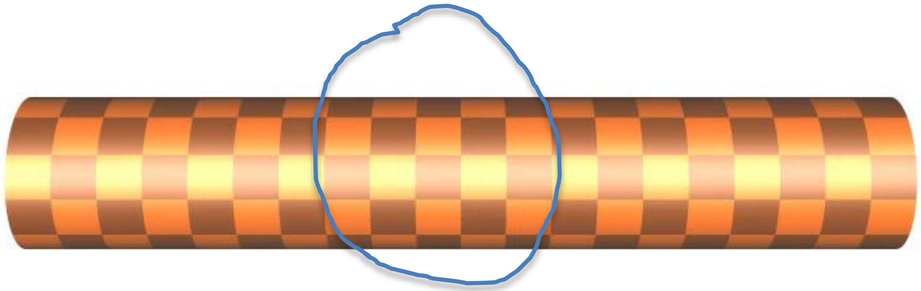
Differential blending [Oztireli et al. 2013]



Dual quaternion bulging artifact



Dual quaternion bulging artifact



Dual quaternion bulging artifact

Not always undesirable (knuckles)

Artists want control

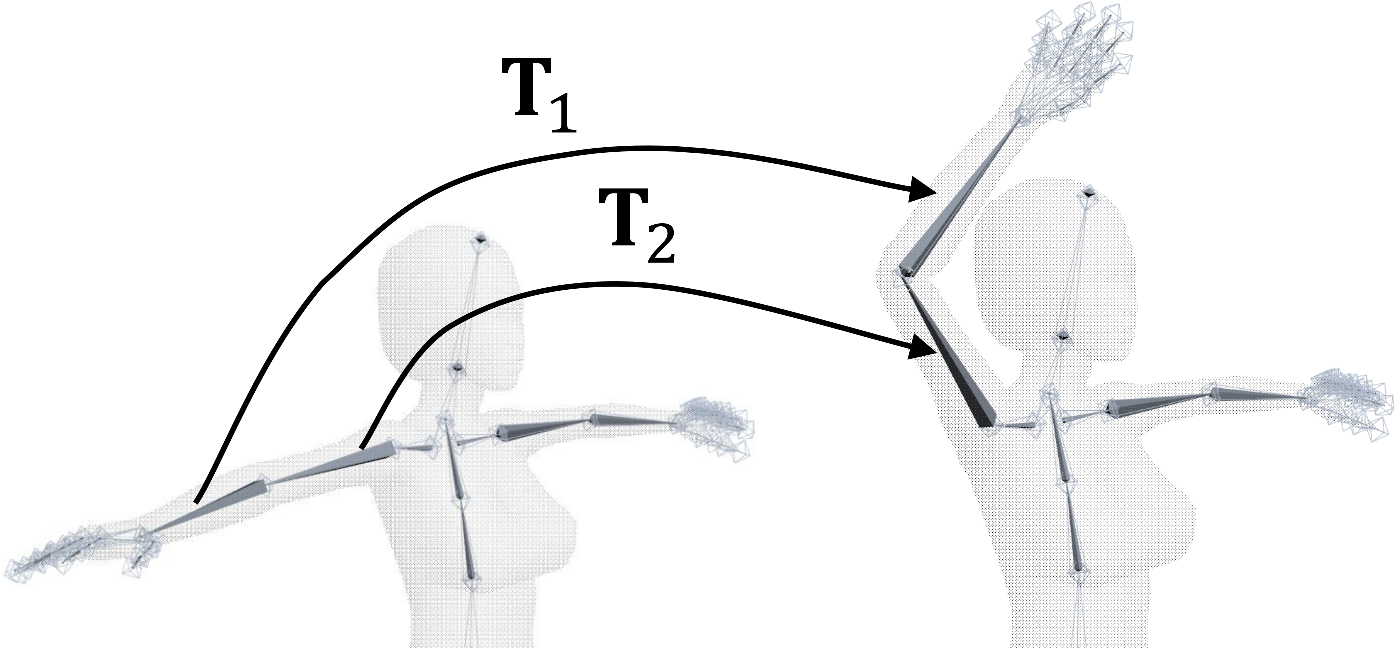
Autodesk Maya: blend LBS and DQS

Finer control is desirable

Advanced deformation primitives (deformers)

Basic building block of deformations

In LBS & DQS: deformers are skinning transformations

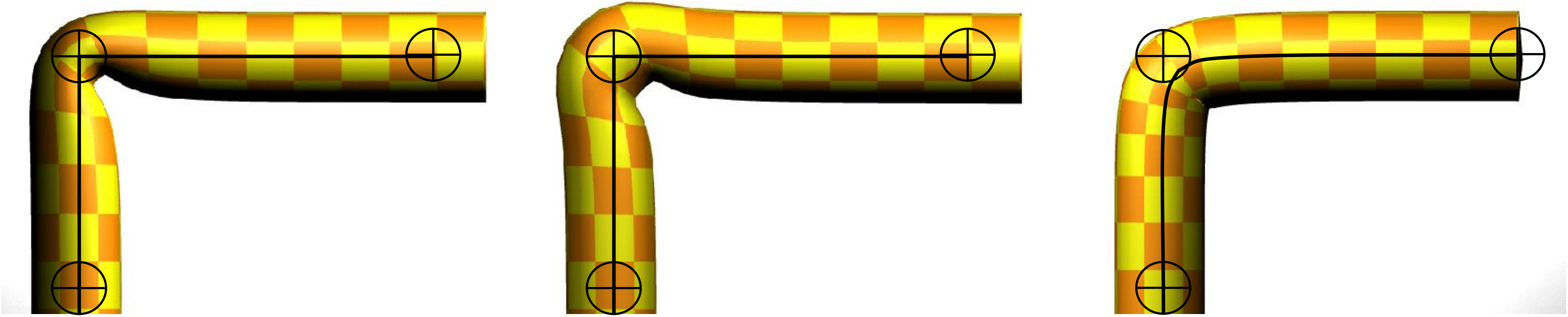


Curve-based deformers

Wires [Singh and Fiume 1998]

[Kalra et al. 1998] [Hyun et al. 2005]

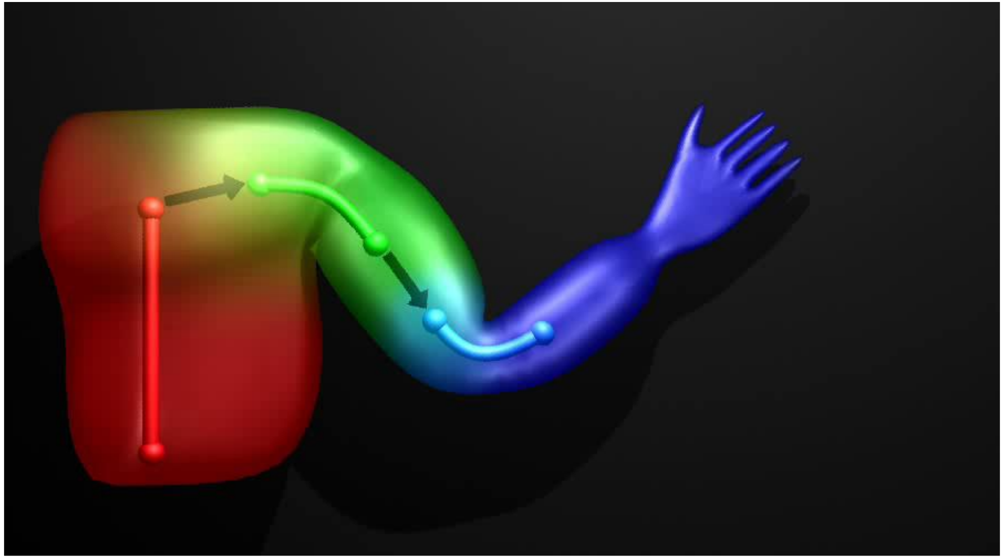
Spline-based skinning [Forstmann et al. 2007]



Linear blending   Dual quaternions   Spline skinning

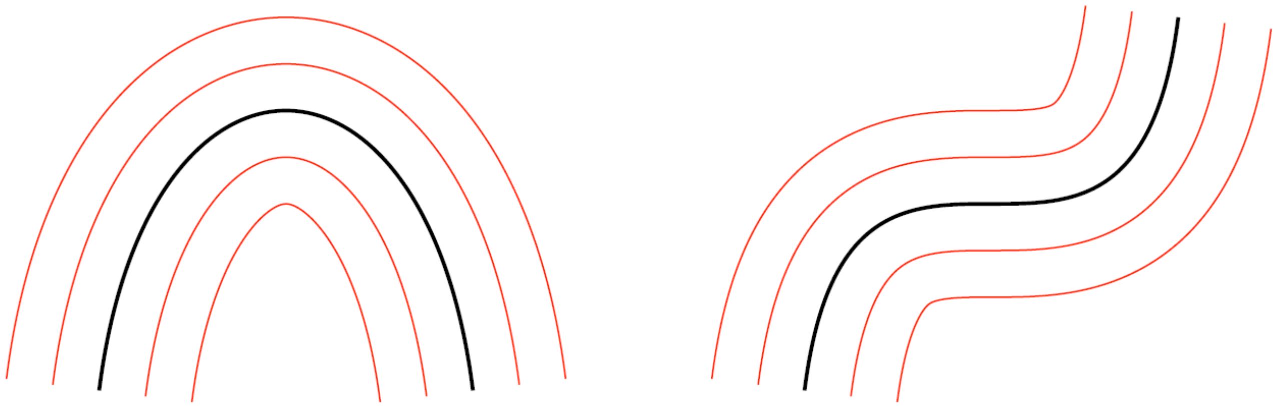
Spline-based skinning [Forstmann et al. 2007]

Deformers blended linearly (like LBS)



Offset curve deformation [Gregory and Weston 2008]

Bind skin to offset curves

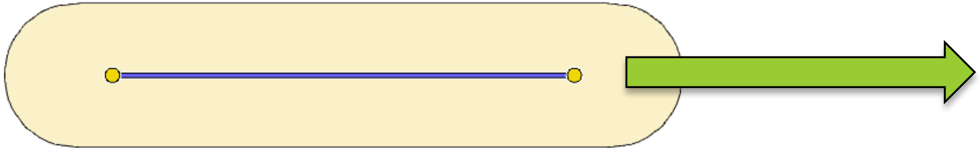


[Sederberg 2012]

Stretchable and twistable bones

[Jacobson and Sorkine 2011]

Standard skinning transforms don't stretch well



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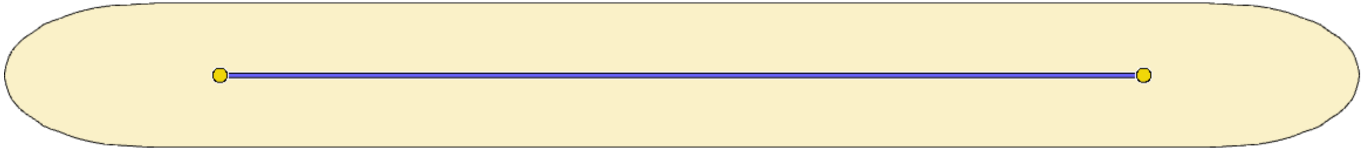
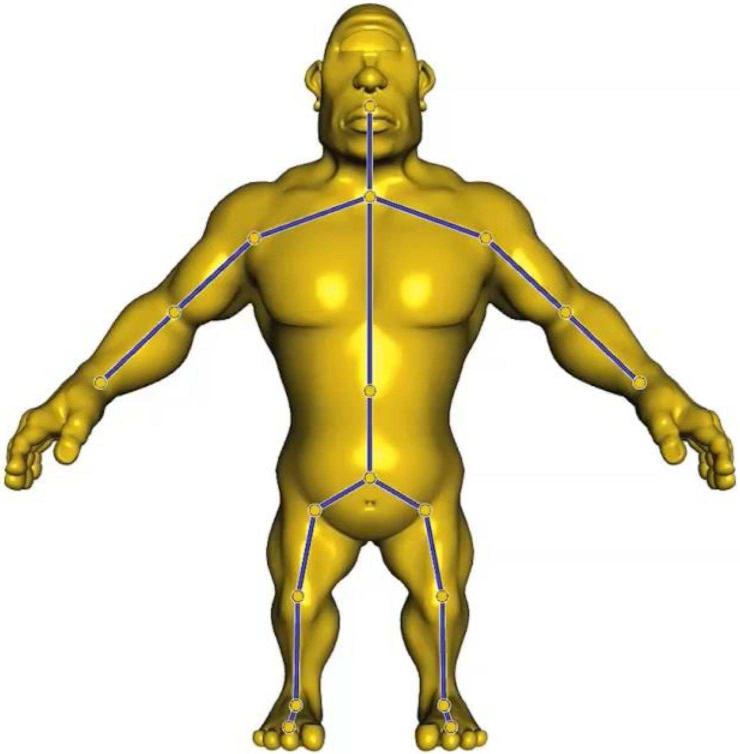
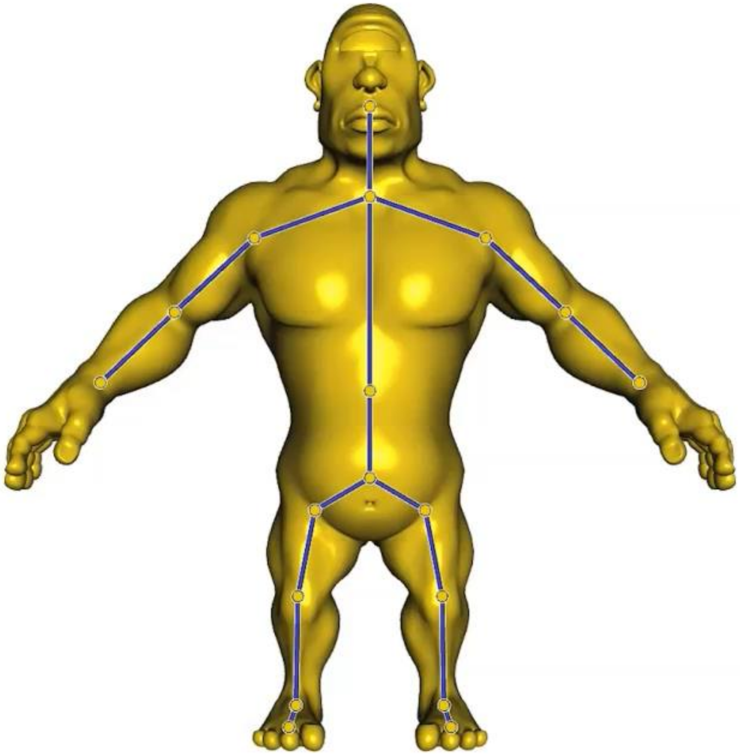


Stretching results in shape explosion

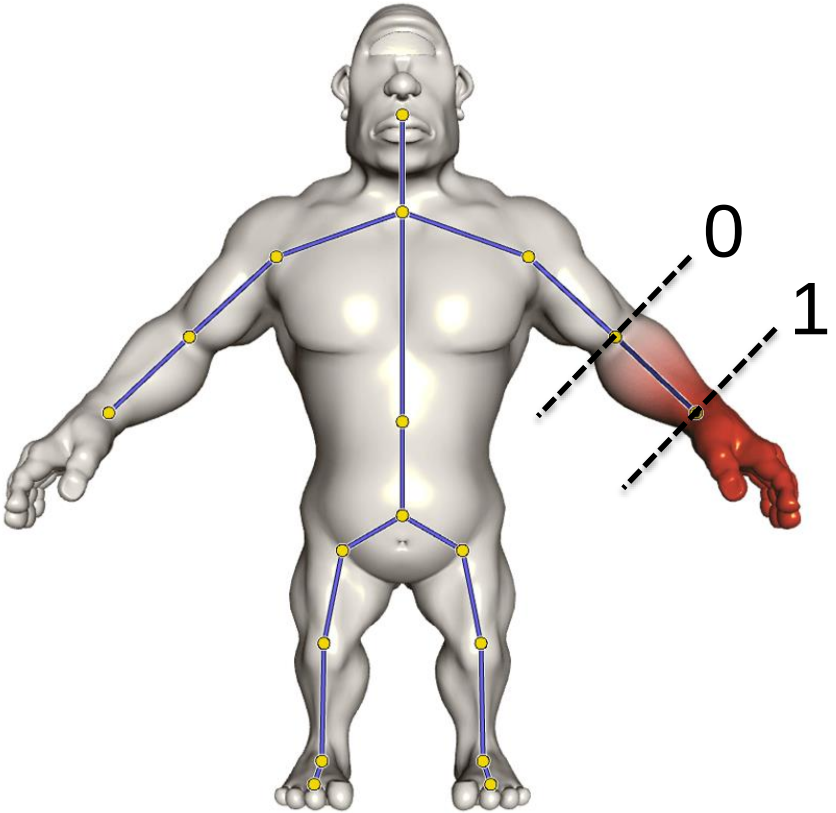


Twisting must be packed at joints



Solution: endpoint weights

[Jacobson and Sorkine 2011]



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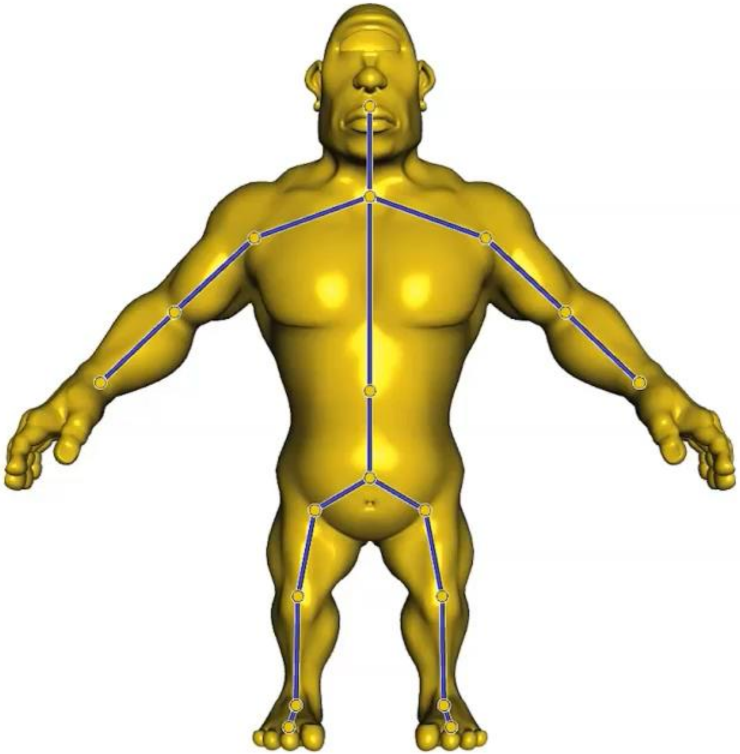

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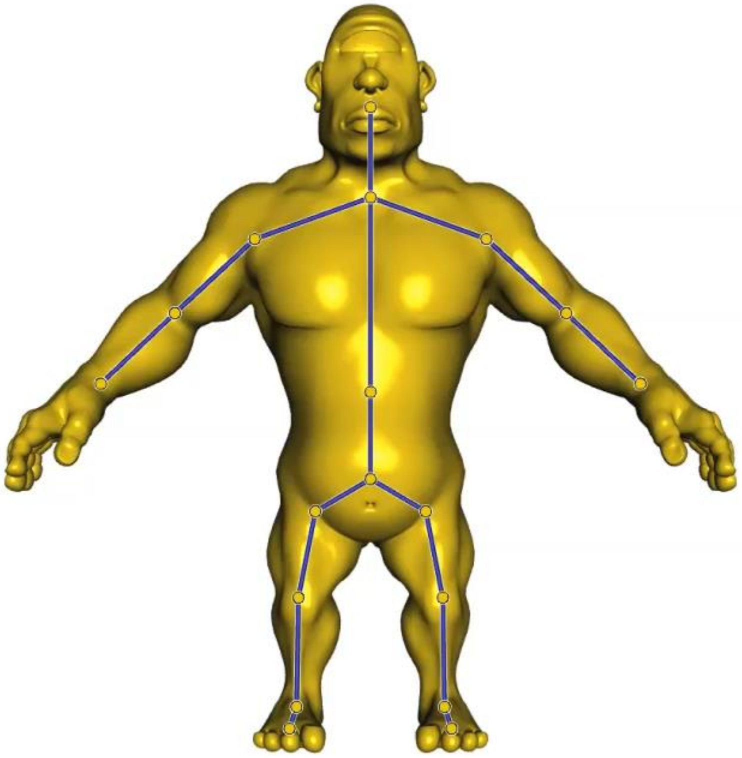
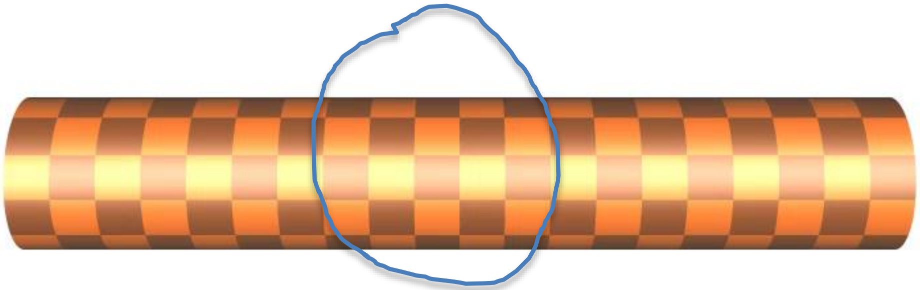
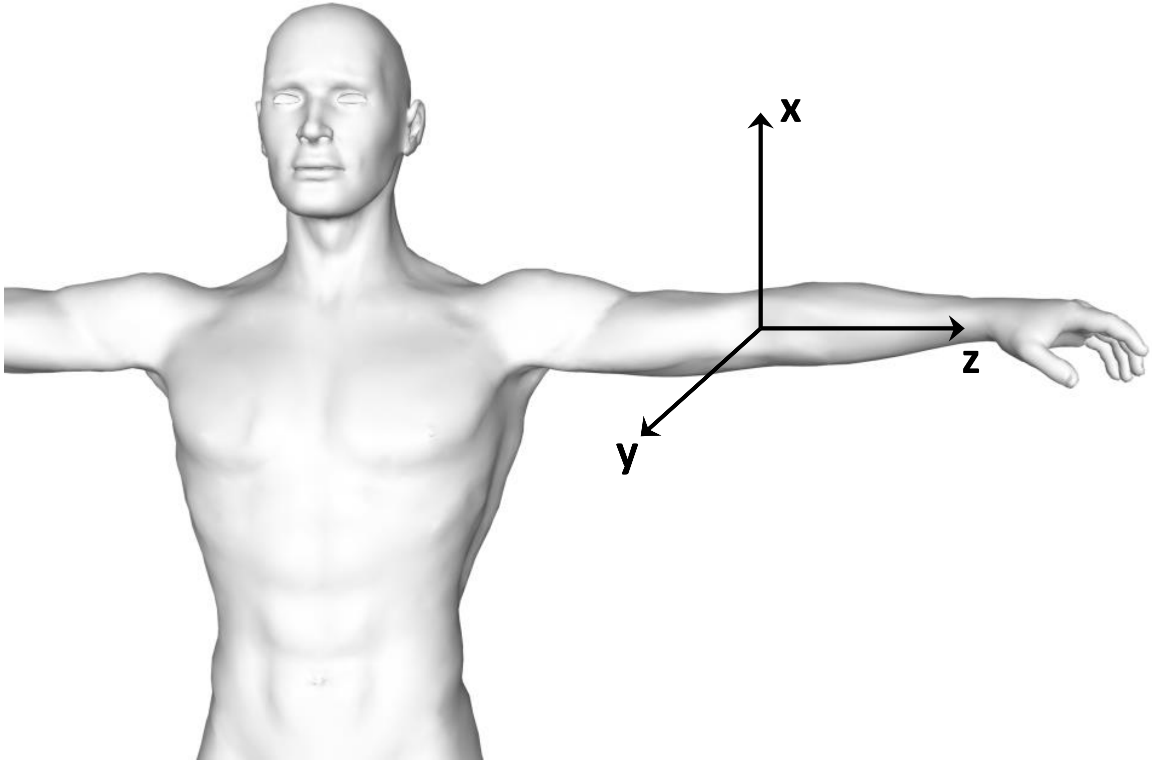


How to fix dual quaternion bulging?



Joint-based deformers [Kavan and Sorkine 2012]



Joint-based deformers [Kavan and Sorkine 2012]

$$R^3 \rightarrow R^3$$

Joint-based deformers [Kavan and Sorkine 2012]

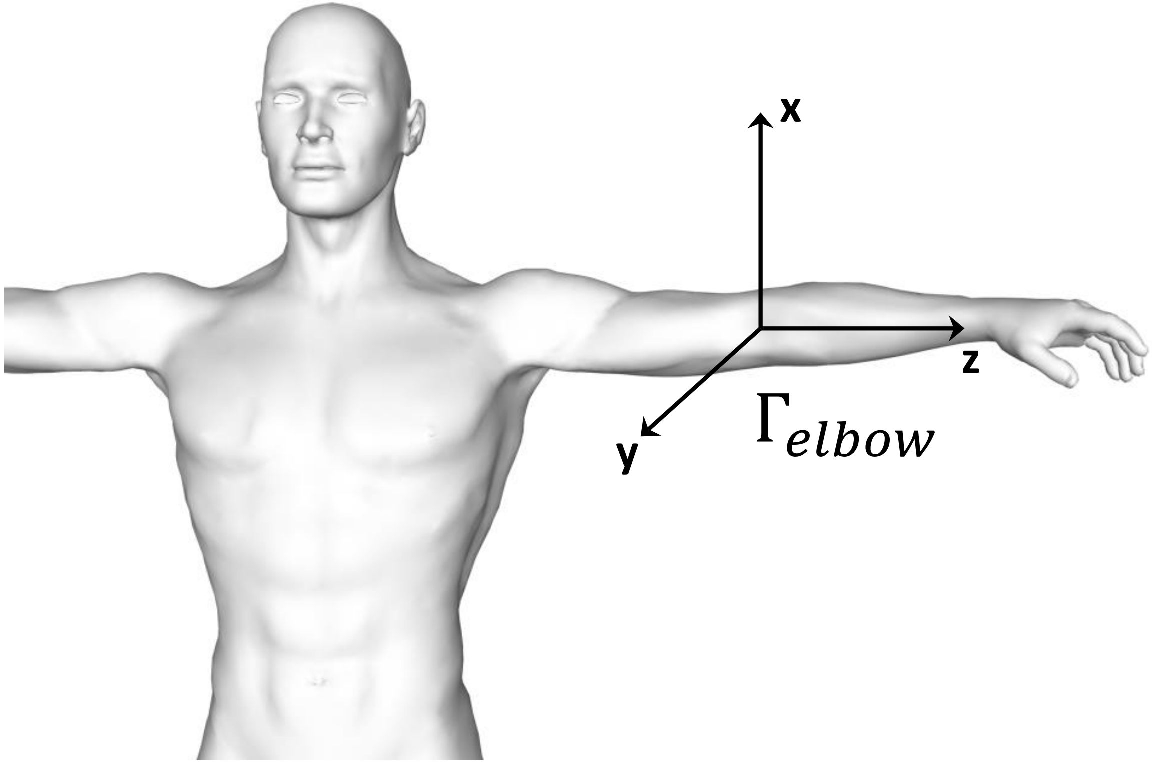
$$SO(3) \times R^3 \to R^3$$

Joint-based deformers [Kavan and Sorkine 2012]

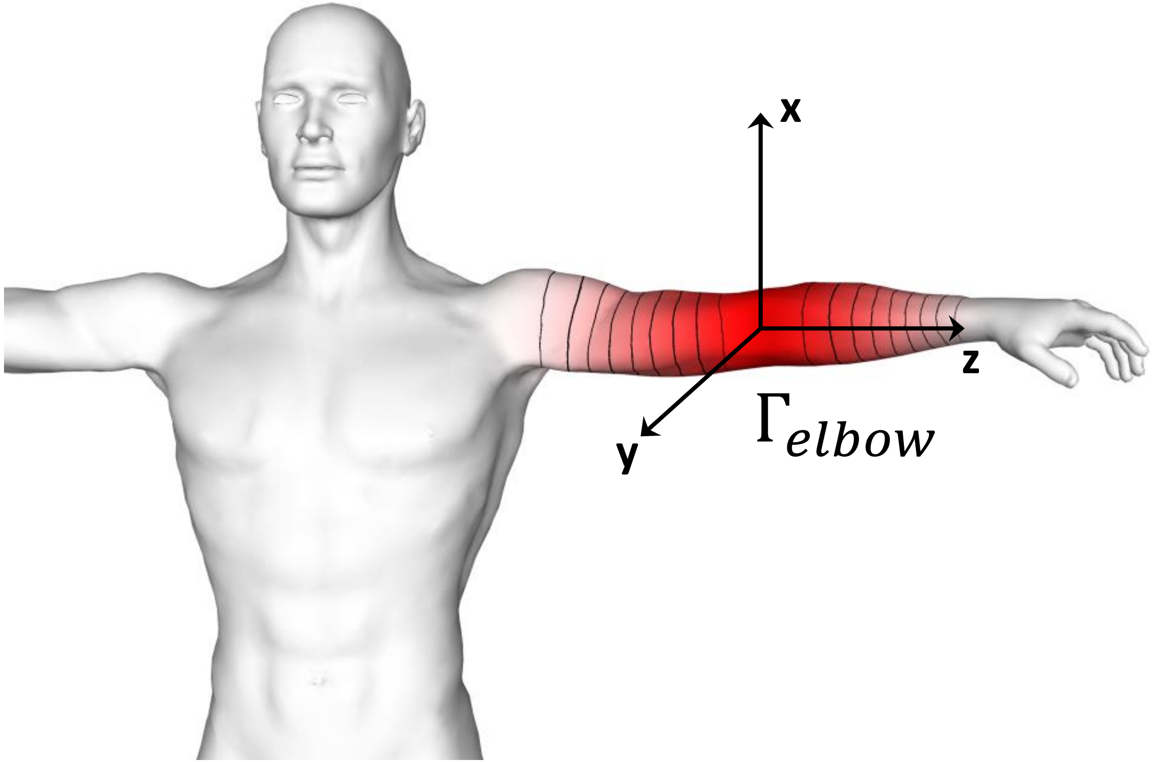
$$\Gamma : SO(3) \times R^3 \rightarrow R^3$$

$$\Gamma(\boldsymbol{Q}, \boldsymbol{x}) = \boldsymbol{x}'$$

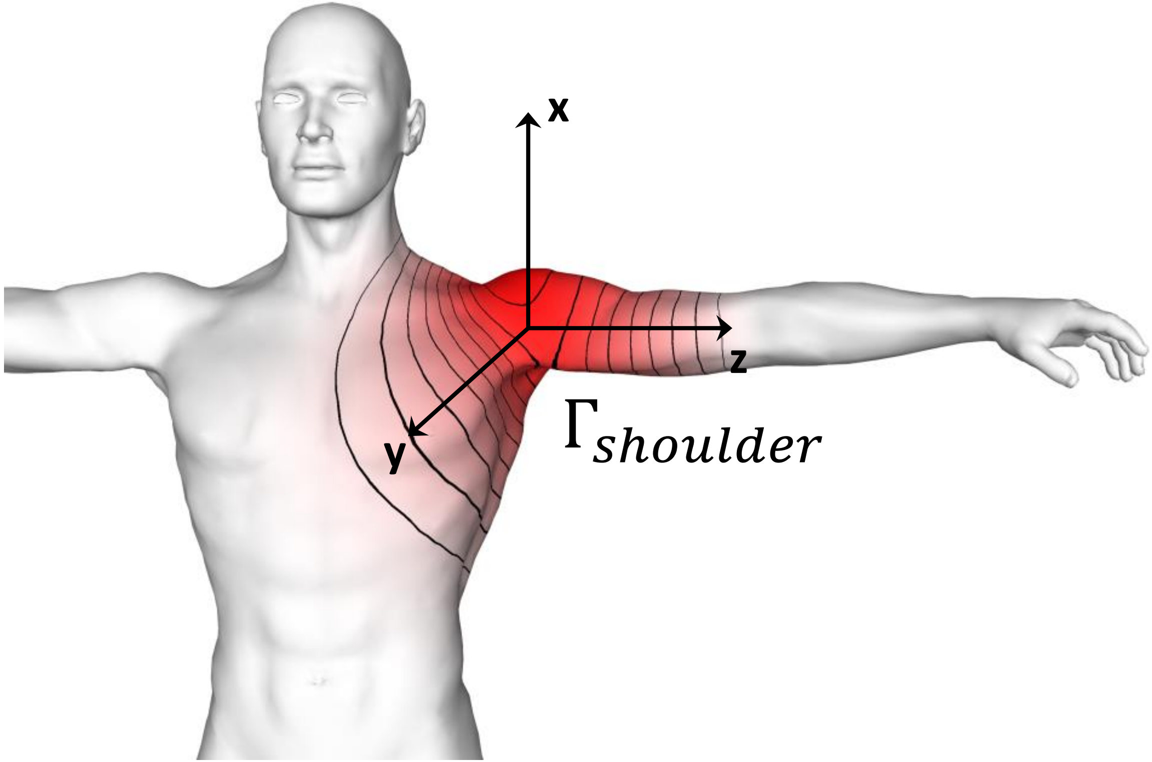
Individual deformers blended linearly



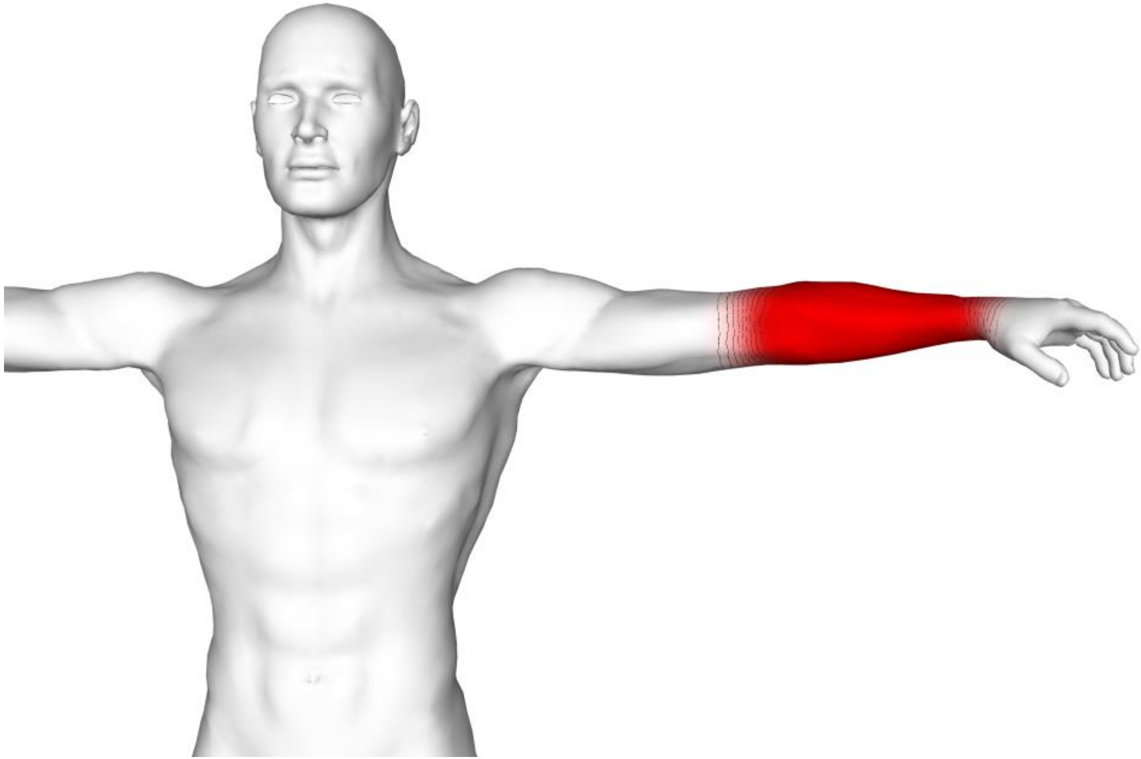
Individual deformers blended linearly



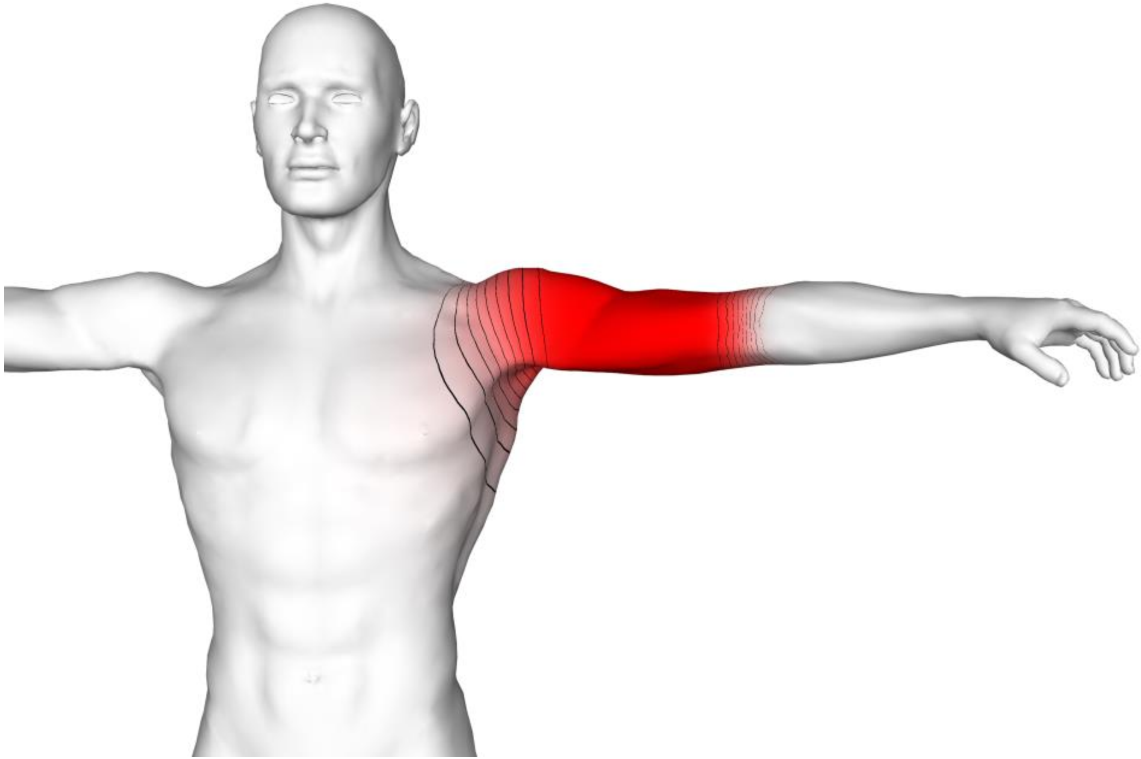
Individual deformers blended linearly



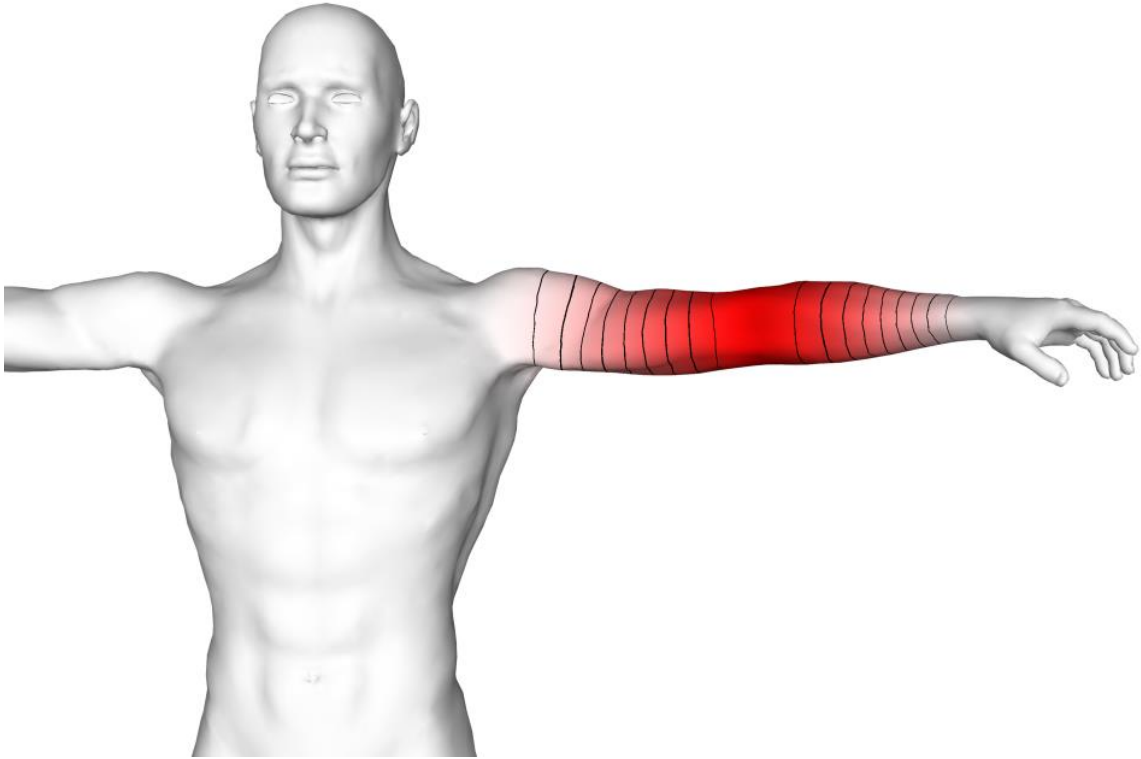
Weights of bone-based deformers



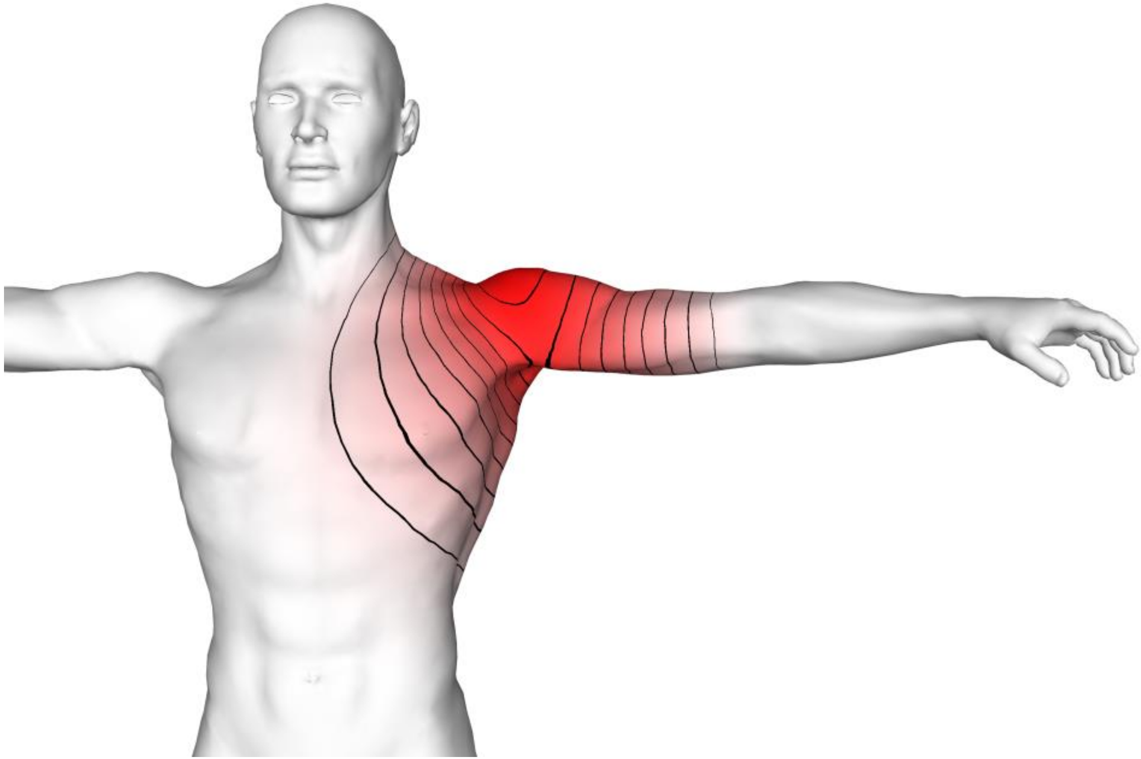
Weights of bone-based deformers



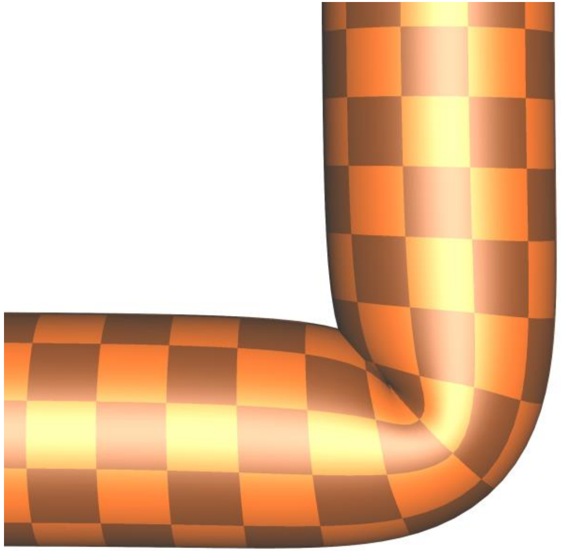
Weights of joint-based deformers



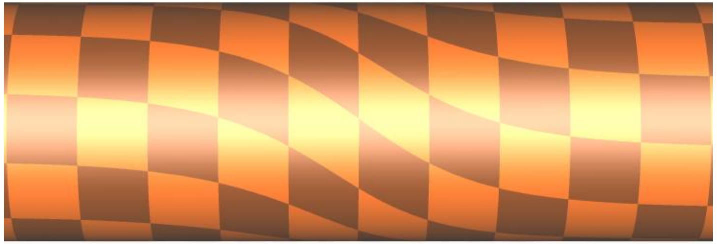
Weights of joint-based deformers



Swing/twist deformer [Kavan and Sorkine 2012]



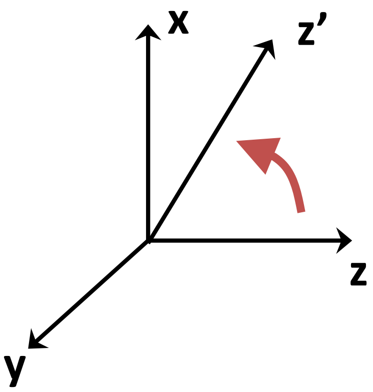
LBS



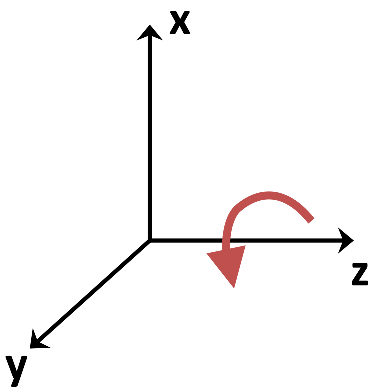
DQS

Swing/twist deformer [Kavan and Sorkine 2012]

$$\Gamma(\boldsymbol{Q}, \boldsymbol{x}) \qquad \boldsymbol{Q} = \boldsymbol{Q}_{swing}\boldsymbol{Q}_{twist}$$



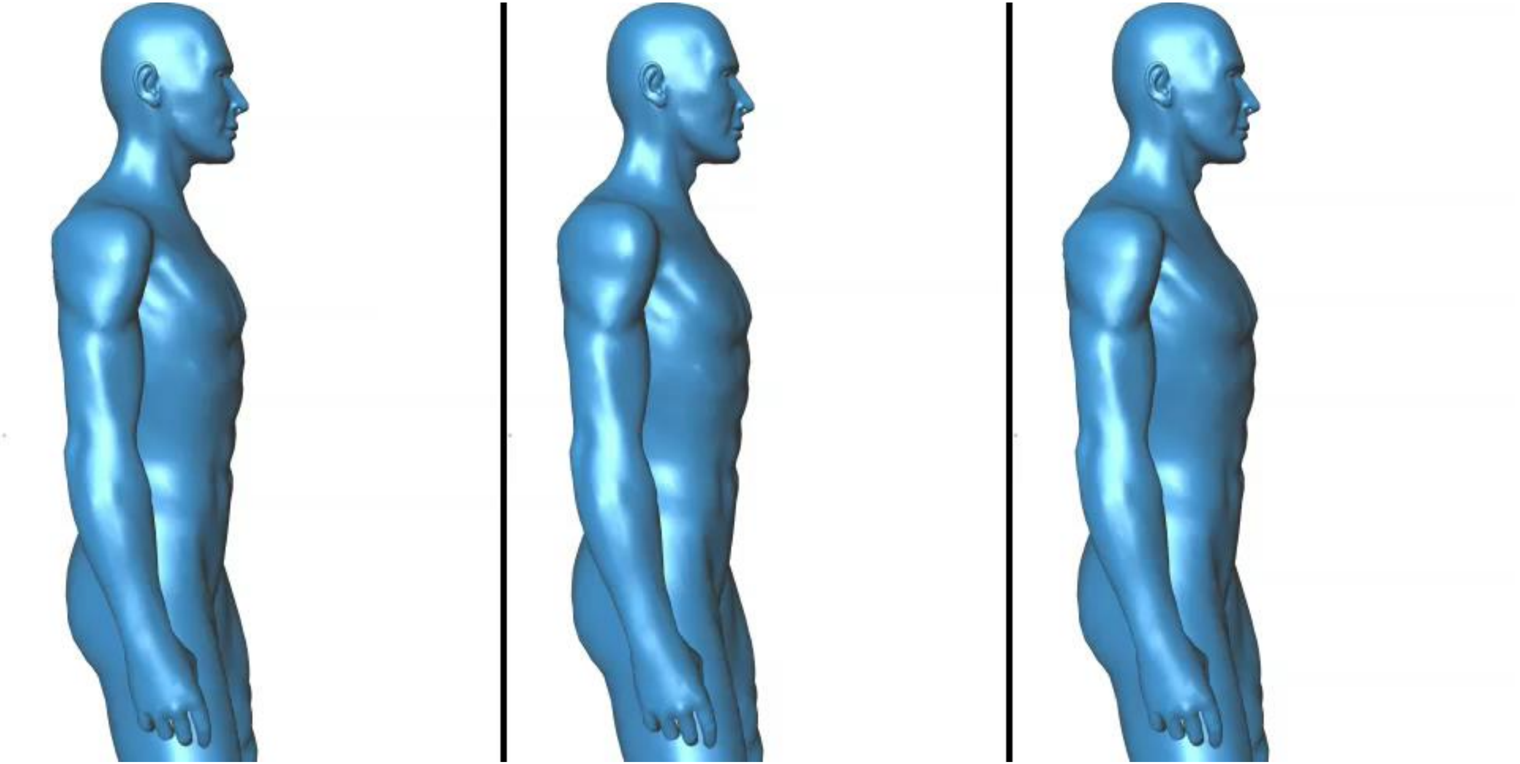
swing



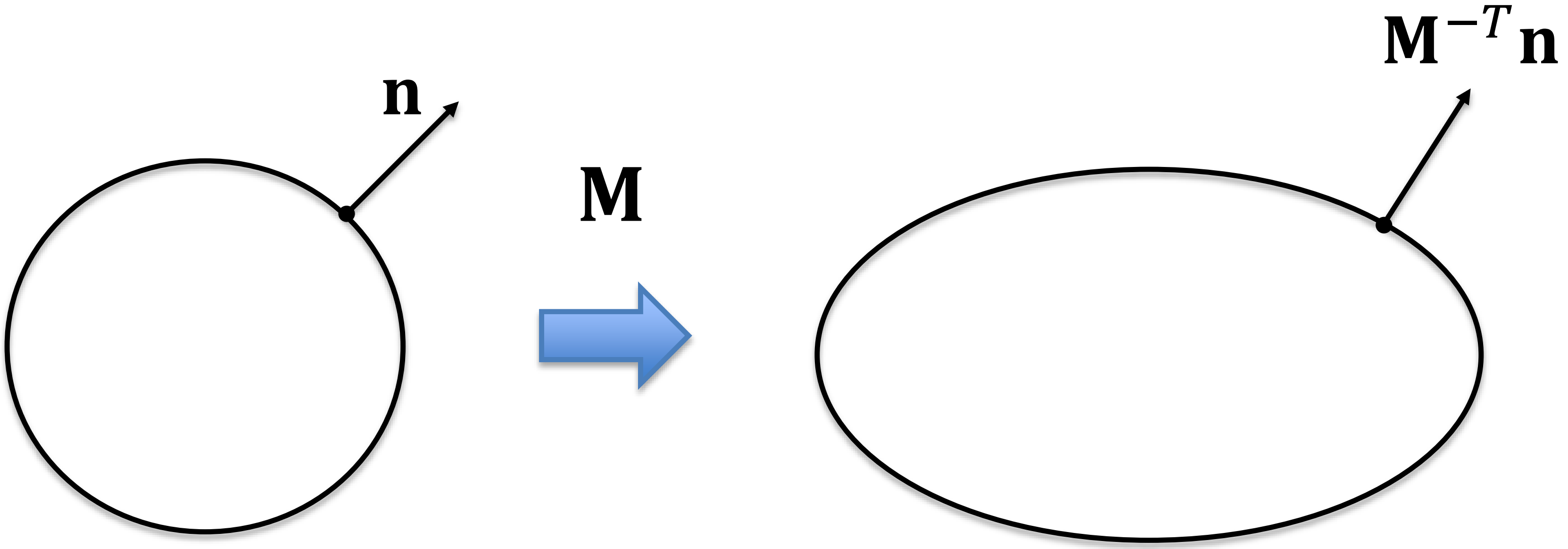
twist

Swing/twist deformer [Kavan and Sorkine 2012]

LBS DQS Swing/twist def.



Skinning normals



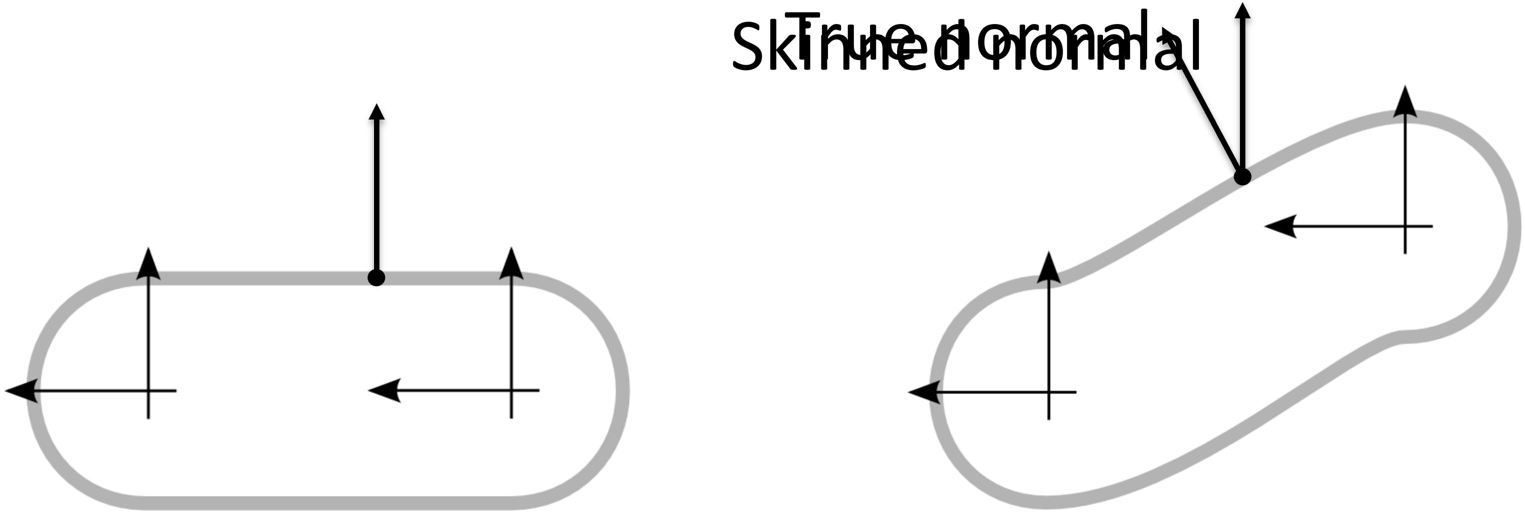
Classical solution

Inverse transpose the linear part of the blended transformation

$$\sum_{j=1}^{m} w_{i,j} \, \mathbf{T}_j$$

This leads to inaccurate normals

[Merry et al. 2006; Tarini et al. 2014]



We can re-compute the normals…

… but that’s not ideal, esp. on the GPUs

Correct skinned normals [LBS&DQS]

[Merry et al. 2006; Tarini et al. 2014]

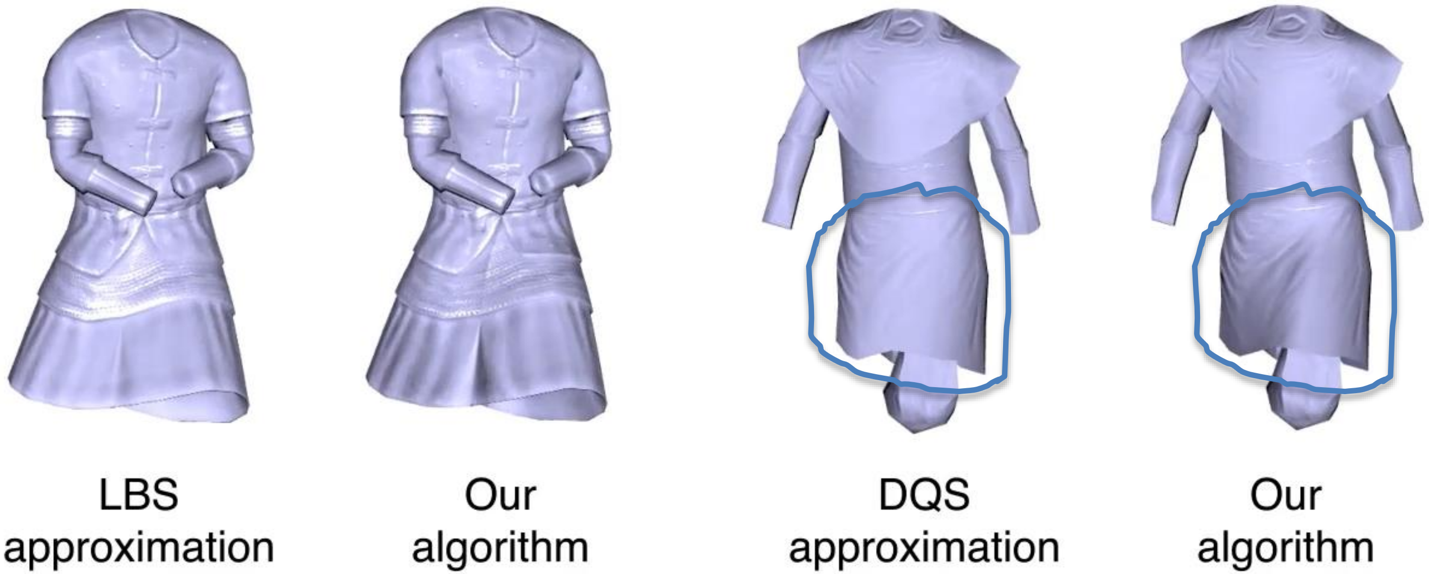
Take weight gradients into account

$$\sum_{j=1}^{m} w_{i,j}\,\mathbf{T}_j + \sum_{j=1}^{m} \mathbf{T}_j \mathbf{v}_i \left(\nabla w_{i,j}\right)^T$$

Inverse transposition can be avoided

Correct skinned normals [LBS&DQS]

[Tarini et al. 2014]



Correct skinned normals [LBS&DQS]

[Tarini et al. 2014]

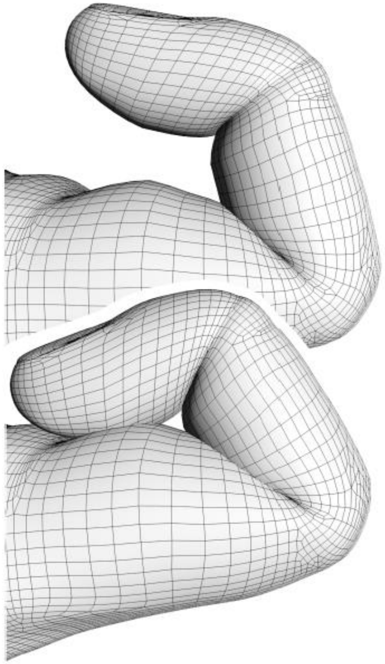
Summary of direct skinning methods

1. Multi-linear methods
2. Nonlinear methods
3. Advanced deformation primitives

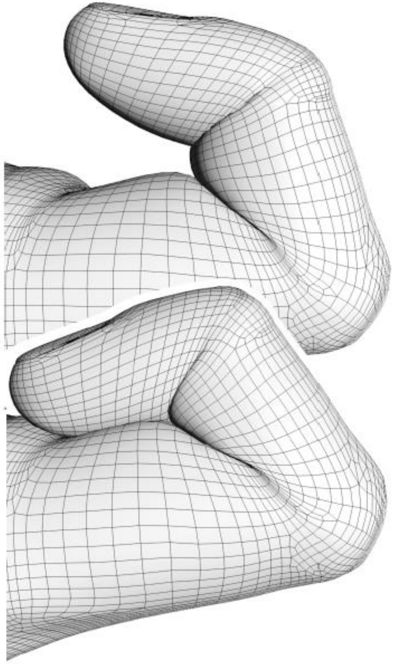
New trends: implicit skinning

[Vaillant et al. 2013]

LBS



Implicit skinning



Course notes: **http://www.skinning.org**

Skinning: Real-time Shape Deformation

ACM SIGGRAPH 2014 Course

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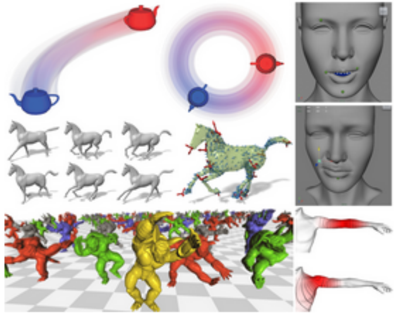

Course Materials

- Part I: Direct methods (Ladislav Kavan)
  Course notes
- Part II: Automatic methods (Alec Jacobson)
  Course notes | Course notes (low resolution)
- Part III: Example-based methods (JP Lewis)
  Course notes (coming soon)
- Part IV: Skinning decomposition (Zhigang Deng)
  Course notes | Course notes (low resolution)