Skinning: Real-time Shape Deformation Automatic Skinning via Constrained Energy Optimization

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 $\mathbf{v}' = \sum_{j \in H} w_j(\mathbf{v}) \mathbf{T}_j \begin{pmatrix} \mathbf{v} \\ 1 \end{pmatrix}$















Recent research automates each step



Recent research automates each step



Skeleton implies shape...



[Blair 1994]

...and shape implies skeleton



[Au et al. 2008]

State-of-the-art skeleton extraction techniques split into two camps



Surface flow degenerates to skeleton approximating medial axis



Adjacency graph of rigid segmentation encodes topology of skeleton



Assume we know which parts will deform rigidly

But which parts will deform rigidly?



Eigenmodes of as-rigid-as-possible energy form a reasonable *prior*



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Eigenmodes of as-rigid-as-possible energy form a reasonable *prior*



Eigenmodes of as-rigid-as-possible energy form a reasonable prior



Skeleton extraction problems remain open

- Joint placement (center of rotation)?
- Guarantee *bones* are inside?
- Respect (intrinsic) symmetries?

Skeleton *embedding* assumes skeletal topology is known



[[]Baran & Popovic 2007]

Categorization of desirable qualities implies energy minimization

- 1. avoid short bone chains,
- 2. maintain angles between bones,
- 3. keep lengths of symmetric bones proportional,
- 4. avoid overlapping bones,
- 5. place special "feet" bones below others,
- 6. avoid zero-length bones,
- 7. maintain bone orientations,
- 8. place extremities close to surface,
- avoid disparities in Euclidean and bone-path distances
 ...



[Baran & Popovic 2007]

Categorization of desirable qualities implies energy minimization



- 8. place extremities close to surface,
- avoid disparities in Euclidean and bone-path distances
 ...

Skeleton embedding problems remain open

- Directly measure implied deformation?
- Partial skeletons?



Similar problems arise for automatic cages

- Fully exterior?
- Tight fitting?
- Symmetric?
- "Well-placed"?



Interpenetrating mesh simplification

Recent research automates each step



Weights should obtain a few basic qualities



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Inverse Euclidean distance weights are too crude, show obvious artifacts





$$w_j(\mathbf{v}) = \frac{1}{\|\mathbf{c}_i - \mathbf{v}\|^2}$$

[Shepard 1968], [Schaefer et al. 2006], etc.

weights optimized *inside* shape

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$$w_j(\mathbf{v}) = \frac{1}{\|\mathbf{c}_i - \mathbf{v}\|^2}$$

weights optimized inside shape

[Shepard 1968], [Schaefer et al. 2006], etc. Generalize barycentric coordinates capture *shape-awareness* via low-resolution "cage"

m $\mathbf{v}' = \sum w_j(\mathbf{v})\mathbf{c}'_j$ i=1

"Coordinate" or reproduction property defines any point in cage as weighted sum of vertices

m $\mathbf{v}' = \sum w_j(\mathbf{v})\mathbf{c}'_j$ $\overline{j=1}$

"coordinates" m $\mathbf{v} = \sum w_j(\mathbf{v})\mathbf{c}_j$ j=1

Coordinate-based deformation is a special case of linear blend skinning

m $\mathbf{v}' = \sum w_j(\mathbf{v})\mathbf{c}'_j$ $\mathbf{v} = \sum w_j(\mathbf{v})\mathbf{c}_j$ $\overline{j=1}$ j=1 $\mathbf{v}' = \sum_{j=1}^{m} w_j(\mathbf{v}) \mathbf{T}_j \begin{pmatrix} \mathbf{v} \\ 1 \end{pmatrix}$ restricted to translation






m $\mathbf{v}' = \sum w_j(\mathbf{v})\mathbf{c}'_j$ m $\mathbf{v} = \sum w_j(\mathbf{v})\mathbf{c}_j$ j=1j=1m $\mathbf{v}' = \sum w_j(\mathbf{v})\mathbf{c}'_j$ $\overline{i=1}$

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"coordinates"
$$\mathbf{v} = \sum_{j=1}^m w_j(\mathbf{v}) \mathbf{c}_j$$

"Higher order GBC" [Langer et al. 2008] are identical to linear blend skinning

Method	N-GON	CONCAVE	SHAPE	≥ 0	C^1	LAG.	CLOSED	Mono.	Out	LOCAL	Poly	COORD.
Barycentric	Х	X	•	•	•	•	•	•	•	Х	Х	•
Wachspress	•	Х	•	Х	•	•	•	Х	?	Х	Х	•
Natural Neighbor	•	•	Х	•	Х	•	•	•	•	•	•	•
Mean Value	•	•	•	Х	•	•	•	Х	•	Х	Х	•
Green and others	•	•	•	Х	•	Х	•	Х	Х	Х	Х	•
Positive Mean Value	•	•	•	•	Х	•	Х	Х	Х	Х	Х	•
Harmonic	•	•	•	•	•	•	Х	•	Х	Х	•	•
Maximum Entropy	•	•	•	•	•	•	Х	?	Х	Х	•	•
Const. Biharmonic	•	•	•	•	•	•	Х	•	Х	•	•	Х

Method	N-gon	CONCAVE	Shape	≥ 0	C^1	LAG.	CLOSED	Mono.	Out	LOCAL	Poly	COORD.
Barycentric	Х	Х	•	•	•	•	•	•	•	Х	Х	•
Wachspress	•	Х	•	Х	•	•	•	Х	?	Х	Х	٠
Natural Neighbor	•	•	Х	•	Х	•	•	•	•	•	•	•
Mean Value	•	•	•	Х	•	•	•	Х	•	Х	Х	•
Green and others	•	•	•	Х	•	Х	•	Х	Х	Х	Х	•
Positive Mean Value	•	•	•	•	Х	•	Х	Х	Х	Х	Х	•
Harmonic	•	•	•	•	•	•	Х	•	Х	Х	•	•
Maximum Entropy	•	•	•	•	•	•	Х	?	Х	Х	•	•
Const. Biharmonic	٠	•	٠	•	•	•	Х	•	Х	•	•	Х

Method	N-gon	CONCAVE	Shape	≥ 0	C^1	LAG.	CLOSED	Mono.	Out	LOCAL	Poly	COORD.
Barycentric	Х	Х	•	•	•	•	•	•	•	Х	Х	•
Wachspress	•	Х	•	Х	•	•	•	Х	?	Х	Х	٠
Natural Neighbor	•	•	Х	•	Х	•	•	•	•	•	•	•
Mean Value	•	•	•	Х	•	•	•	Х	•	Х	Х	•
Green and others	•	•	•	Х	•	Х	•	Х	Х	Х	Х	•
Positive Mean Value	•	•	•	•	Х	•	Х	Х	Х	Х	Х	•
Harmonic	•	•	•	•	•	•	X	•	Х	Х	•	•
Maximum Entropy	•	•	•	•	•	•	Х	?	Х	Х	•	•
Const. Biharmonic	٠	•	•	•	•	•	X	٠	Х	•	•	Х

Method	N-GON	CONCAVE	SHAPE	≥ 0	C^1	LAG.	CLOSED	Mono.	Out	LOCAL	Poly	COORD.
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Natural Neighbor	•	•	Х	•	Х	•	•	•	•	•	•	•
Mean Value	•	•	•	Х	•	•	•	Х	•	Х	Х	•
Green and others	•	•	•	Х	•	Х	•	Х	Х	Х	Х	•
Positive Mean Value	•	•	•	•	Х	•	Х	Х	Х	Х	Х	•
Harmonic	•	•	•	•	•	•	Х	•	Х	Х	•	•
Maximum Entropy	•	•	•	•	•	•	Х	?	Х	Х	•	•
Const. Biharmonic	•	•	•	•	•	•	Х	•	Х	•	•	Х

No free lunch? Are some properties mutually exclusive?











[Baran & Popović 2007]



 $\underset{w_j}{\operatorname{argmin}} \int_{\Omega} \|\nabla w_j\|^2 + h_j (w_j - \hat{w}_j)^2 \, dA$

 \mathcal{U}







 $\underset{w_j}{\operatorname{argmin}} \int_{\Omega} \left\| \nabla w_j \right\|^2 + h_j (w_j - \hat{w}_j)^2 \, dA$

W

Gradient energy weights not smooth at handles



Gradient energy weights not smooth at handles





Gradient energy weights not smooth at handles



$$\Delta^2 w_j = 0$$



 $\Delta w_j = 0$

Any single handle type is too restrictive...







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Each handle type has a specific task, more than just a *different modeling metaphor*



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Non-negative, local weights are mandatory



Non-negative, local weights are mandatory $0 \le w_j \le 1$ 0 $\int_{\Omega} (\Delta w_j)^2 dA$ argmin 0 (• w_i [Botsch & Kobbelt 2004] 0

Spurious extrema cause distracting artifacts





Must explicitly prohibit spurious extrema





Previous methods fail in one way or another

<u>:</u>	Euclidean	Δw_j	$\Delta^2 w_j$
smooth	\checkmark	-	\checkmark
non-negative	\checkmark	\checkmark	-
shape-aware	-	\checkmark	\checkmark
local	-/ 🗸	-	-
monotonic	-	1	-
arbitrary handles	-	\checkmark	\checkmark
	[Shepard 1968, Sibson 1980, Schaefer et al. 2006]	[Baran & Popovic 2007, Joshi et al. 2007]	[Botsch & Kobbelt 2004, Sorkine et al. 2004, Finch et al. 2011]
No free lunch? Are some properties m	nutually exclusive?		

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<u>-</u>	Euclidean	Δw_j	$\Delta^2 w_j$
smooth	\checkmark	-	\checkmark
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monotonic	-	\checkmark	-
arbitrary handles	-	\checkmark	\checkmark
	[Shepard 1968, Sibson 1980, Schaefer et al. 2006]	[Baran & Popovic 2007, Joshi et al. 2007]	[Botsch & Kobbelt 2004, Sorkine et al. 2004, Finch et al. 2011]
No free lunch? Are some properties mu	utually exclusive?		

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+

$$\underset{w_j,j=1,\ldots,m}{\operatorname{argmin}} \sum_{j=1}^m \int_{\Omega} (\Delta w_j)^2 \, dV$$

+ shape-aware + smoothness

[Botsch & Kobbelt 2004, Sorkine et al. 2004, Joshi & Carr 2008, Jacobson et al. 2010, Finch et al. 2011, Andrews et al. 2011]

 $\underset{w_j, j=1,...,m}{\operatorname{argmin}} \sum_{i=1}^{m} \int_{\Omega} (\Delta w_j)^2 \, dV$ $w_j(\mathbf{v}) = \begin{cases} 1\\ 0\\ \text{linear on cage facets} \end{cases}$ $\mathbf{v} \in h_j, \\ \mathbf{v} \in h_k$

+ shape-aware+ smoothness+ arbitrary handles

[Botsch & Kobbelt 2004, Sorkine et al. 2004, Joshi & Carr 2008, Jacobson et al. 2010, Finch et al. 2011, Andrews et al. 2011]

$$\underset{w_j, j=1, \dots, m}{\operatorname{argmin}} \sum_{j=1}^m \int_{\Omega} (\Delta w_j)^2 \, dV$$

$$0 \le w_j \le 1,$$

$$\sum_{j=1}^{m} w_j = 1$$

+ shape-aware
+ smoothness
+ arbitrary handles
+ non-negativity

$$\underset{w_j, j=1, \dots, m}{\operatorname{argmin}} \sum_{j=1}^m \int_{\Omega} (\Delta w_j)^2 \, dV$$

$$0 \le w_j \le 1,$$

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[Jacobson et al. 2011]

+ shape-aware
+ smoothness
+ arbitrary handles
+ non-negativity
+ locality

$$\underset{w_j, j=1,...,m}{\operatorname{argmin}} \sum_{j=1}^m \int_{\Omega} (\Delta w_j)^2 \, dV$$

 $||w||_1 = 1$

+ shape-aware
+ smoothness
+ arbitrary handles
+ non-negativity
+ locality
$$\underset{w_j, j=1, \dots, m}{\operatorname{argmin}} \sum_{j=1}^m \int_{\Omega} (\Delta w_j)^2 \, dV$$

$$||w||_1 = 1 \to \sum_{j=1}^m |w_j| = 1$$

+ shape-aware
+ smoothness
+ arbitrary handles
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+ locality

 $\underset{w_j,j=1,\ldots,m}{\operatorname{argmin}} \sum_{j=1}^m \int_{\Omega} (\Delta w_j)^2 \, dV$

$$||w||_{1} = 1 \to \sum_{j=1}^{m} |w_{j}| = 1 \to \sum_{j=1}^{m} w_{j} = 1,$$
$$0 \le w_{j} \le 1$$

+ shape-aware
+ smoothness
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+ locality

[Rustamov 2011]

$$\underset{w_j, j=1, \dots, m}{\operatorname{argmin}} \sum_{j=1}^{m} \int_{\Omega} (\Delta w_j)^2 \, dV$$

$$\nabla w_j \cdot \nabla u_j > 0$$

+ shape-aware
+ smoothness
+ arbitrary handles
+ non-negativity
+ locality
+ monotonicity

[Weinkauf et al. 2011, Jacobson et al. 2012, Günther et al. 2014]

Previous methods fail in one way or another

-	Euclidean	Δw_j = u	$\Delta^2 w_j$
smooth	\checkmark	-	\checkmark
non-negative	\checkmark	\checkmark	_
shape-aware	-	\checkmark	\checkmark
local	-/ 🗸	_	_
monotonic	-	1	-
arbitrary handles	_	\checkmark	\checkmark
	[Shepard 1968, Sibson 1980, Schaefer et al. 2006]	[Baran & Popovic 2007, Joshi et al. 2007]	[Botsch & Kobbelt 2004, Sorkine et al. 2004, Finch et al. 2011]



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Weights retain nice properties in 3D



Weights retain nice properties in 3D



Variational formulation allows additional, problem-specific constraints



Variational formulation allows additional, problem-specific constraints





Energy encodes *prior* on deformation, could swap smoothness for rigidity

Left, middle: smoothness energy

Right: [Kavan & Sorkine 2012]

Energy encodes *prior* on deformation, could swap smoothness for rigidity

Left, middle: smoothness energy

Right: [Kavan & Sorkine 2012]

Self-intersections
Nonmanifold edges
Open boundaries

Multiple connected components







[Dionne & de Lasa 2013]



Recent research automates each step



Good weights are not enough



Rotation chosen ignorant of shape's deformation



Rotation *optimized* for best deformation [Jacobson et al. 2011]

Good weights are not enough



Rotation chosen ignorant of shape's deformation [Jacobson et al. 2011], cf. traditional IK



Rotation *optimized* for best deformation [Jacobson et al. 2011]

Good weights are not enough



Rotation chosen ignorant of shape's deformation [Jacobson et al. 2011], cf. traditional IK



Rotation *optimized* for best deformation [Jacobson et al. 2011]

Shape-aware IK finds *best* transformations as measured by shape's deformation



Shape-aware IK finds *best* transformations as measured by shape's deformation





full space: #V x 3 DOFs

[traditional physics, modeling]





skinning space: #B x 12 DOFs

[Der et al. 2006, Huang et al. 2006, Au et al. 2007, Jacobson et al. 2012]



modal space: 50 linear modes

[Hildebrandt et al. 2011, Barbic et al. 2012]



inherits

semantics

from rig

Regularity and parsimony allows *very fast* non-linear energy optimization at runtime

[Jacobson et al. 2012]

Same idea applies for real-time physically based dynamics



[Faure et al. 2011, Jacobson 2013, Liu et al. 2013, Bouaziz et al. 2014]



Posing still requires more research

- Safe contacts, collisions in real time
- Leverage amassed data: semantics
- Better virtual and physical interfaces

Recent research automates each step



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